Help – HW 5

Solution or Explanation

(a) If $p_{\text{ball}} = p_{\text{bullet}}$, then

$$v_{\text{ball}} = \frac{m_{\text{bullet}} v_{\text{bullet}}}{m_{\text{ball}}} = \frac{(3.04 \times 10^{-3}\ \text{kg})(1.50 \times 10^{3}\ \text{m/s})}{0.146\ \text{kg}} = 31.2\ \text{m/s}$$

(b) The kinetic energy of the bullet is

$$KE_{\text{bullet}} = \frac{1}{2} m_{\text{bullet}} v^2_{\text{bullet}} = \frac{(3.04 \times 10^{-3}\ \text{kg})(1.50 \times 10^{3}\ \text{m/s})^2}{2} = 3.42 \times 10^{3}\ \text{J}$$

while that of the baseball is

$$KE_{\text{ball}} = \frac{1}{2} m_{\text{ball}} v^2_{\text{ball}} = \frac{(0.146\ \text{kg})(31.2\ \text{m/s})^2}{2} = 71.2\ \text{J}.$$

The bullet has the larger kinetic energy by a factor of 48.0.

(a) The soccer ball's momentum change is $\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i$. Here, the soccer ball is initially at rest with $\vec{\mathbf{p}}_i = 0$ so that $p_{i,x} = p_{i,y} = 0$.

The x- and y-component of the momentum change are the following.

$$\begin{aligned} \Delta p_x &= p_{f,x} - p_{i,x} \\ &= mv_x - 0 \\ &= (0.425\ \text{kg})(16.0\ \text{m/s}) \cos(37.0°) \\ &= 5.43\ \text{kg} \cdot \text{m/s} \end{aligned}$$

$$\begin{aligned} \Delta p_y &= p_{f,y} - p_{i,y} \\ &= mv_y - 0 \\ &= (0.425\ \text{kg})(16.0\ \text{m/s}) \sin(37.0°) \\ &= 4.09\ \text{kg} \cdot \text{m/s} \end{aligned}$$

(b) From the impulse-momentum theorem, the magnitude of the average force is $F_{\text{av}} = \frac{|\Delta\vec{\mathbf{p}}|}{\Delta t}$ where Δt is the time interval the player's foot is in contact with the ball. Substitute known values to find the following.

$$\begin{aligned} F_{\text{av}} &= \frac{|\Delta\vec{\mathbf{p}}|}{\Delta t} \\ &= \frac{\sqrt{(\Delta p_x)^2 + (\Delta p_y)^2}}{\Delta t} = \frac{\sqrt{(5.43\ \text{kg} \cdot \text{m/s})^2 + (4.09\ \text{kg} \cdot \text{m/s})^2}}{0.0550\ \text{s}} \\ &= 124\ \text{N} \end{aligned}$$

Solution or Explanation

Take the direction of the ball's final velocity (toward the net) to be the $+x$-direction.

(a) $I = \Delta p = m(v_f - v_i) = (0.0600\text{ kg})[39.8\text{ m/s} - (-55.6\text{ m/s})]$ giving $I = +5.72\text{ kg}\cdot\text{m/s} = 5.72\text{ N}\cdot\text{s}$ in direction of final velocity.

(b)
$$W_{net} = \Delta KE = \frac{1}{2}m(v_f^2 - v_i^2)$$
$$= \frac{(0.0600\text{ kg})[(39.8\text{ m/s})^2 - (55.6\text{ m/s})^2]}{2} = -45.2\text{ J}$$

Solution or Explanation

(a) The mass of the rifle is

$$m = \frac{w}{g} = \frac{40\text{ N}}{9.80\text{ m/s}^2} = \left(\frac{40}{9.8}\right)\text{ kg}.$$

We choose the direction of the bullet's motion to be negative. Then, conservation of momentum gives

$$(m_{rifle}v_{rifle} + m_{bullet}v_{bullet})_f = (m_{rifle}v_{rifle} + m_{bullet}v_{bullet})_i$$

or $[(40/9.8)\text{ kg}]v_{rifle} + (4.7 \times 10^{-3}\text{ kg})(-290\text{ m/s}) = 0 + 0$

and $v_{rifle} = \dfrac{9.8(4.7 \times 10^{-3}\text{ kg})(290\text{ m/s})}{40\text{ kg}} = 0.334\text{ m/s}.$

(b) The mass of the man plus rifle is

$$m = \frac{665\text{ N}}{9.80\text{ m/s}^2} = 67.9\text{ kg}.$$

We use the same approach as in (a), to find

$$v = \left(\frac{4.7 \times 10^{-3}\text{ kg}}{67.9\text{ kg}}\right)(290\text{ m/s}) = 2.01 \times 10^{-2}\text{ m/s}.$$

Solution or Explanation

After the 1.70-kg squid has taken in 0.115 kg of water, the squid-water system is stationary and has no momentum. Because no external forces act on the system, its momentum is conserved and, using subscripts s and w for quantities associated with the squid and the water,

$$p_i = p_f$$
$$0 = m_s v_s + m_w v_w$$
$$v_s = -\frac{m_w}{m_s}v_w = -\frac{0.115\text{ kg}}{1.70\text{ kg}}\left(3.00\text{ m/s}\right)$$
$$v_s = -0.203\text{ m/s}$$

The squid achieves a speed of 0.203 m/s.

Solution or Explanation

For the system composed of the three skaters, the momentum vector $\vec{\mathbf{p}}$ is conserved so that $p_{ix} = p_{fx}$ and $p_{iy} = p_{fy}$. The skaters are initially at rest so that $p_{ix} = p_{iy} = 0$. Skaters A and B have final velocity components of $v_{Ax} = 0$, $v_{Ay} = -4.00$ m/s, $v_{Bx} = -3.50$ m/s, and $v_{By} = 0$. Substitute those values to find the components of skater C's final velocity.

$$\begin{aligned} p_{ix} &= p_{fx} \\ 0 &= m_B v_B + m_C v_{Cx} \\ v_{Cx} &= -\frac{m_B}{m_C} v_B \\ &= -\frac{70.0\text{ kg}}{85.0\text{ kg}}(-3.50\text{ m/s}) \\ &= 2.88\text{ m/s} \end{aligned}$$

$$\begin{aligned} p_{iy} &= p_{fy} \\ 0 &= m_A v_A + m_C v_{Cy} \\ v_{Cy} &= -\frac{m_A}{m_C} v_A \\ &= -\frac{75.0\text{ kg}}{85.0\text{ kg}}(-4.00\text{ m/s}) \\ &= 3.53\text{ m/s} \end{aligned}$$

Solution or Explanation

Consider a system consisting of arrow and target from the instant just before impact until the instant after the arrow emerges from the target. No external horizontal forces act on the system, so total horizontal momentum must be conserved, or

$$(m_a v_a + m_t v_t)_f = (m_a v_a + m_t v_t)_i.$$

Thus,

$$\begin{aligned} (v_a)_f &= \frac{m_a (v_a)_i + m_t (v_t)_i - m_t (v_t)_f}{m_a} \\ &= \frac{(22.5\text{ g})(+42.0\text{ m/s}) + (300\text{ g})(-2.15\text{ m/s}) - 0}{22.5\text{ g}} = 13.3\text{ m/s}. \end{aligned}$$

Solution or Explanation

Consider conservation of momentum in the first event (twin A tossing the pack), taking the direction of the velocity given the backpack as positive. This yields

$$m_A v_{Af} + m_{pack} v_{pack} = (m_A + m_{pack})(0) = 0$$

or

$$v_{Af} = \frac{-m_{pack} v_{pack}}{m_A} = -\left(\frac{12.0\text{ kg}}{53.0\text{ kg}}\right)(+2.85\text{ m/s}) = -0.645\text{ m/s} \quad \text{and} \quad |v_{Af}| = 0.645\text{ m/s}.$$

Conservation of momentum when twin B catches and holds onto the backpack yields

$$(m_B + m_{pack}) v_{Bf} = m_B(0) + m_{pack} v_{pack}$$

or

$$v_{Bf} = \frac{m_{pack} v_{pack}}{m_B + m_{pack}} = \frac{(12.0\text{ kg})(+2.85\text{ m/s})}{53.0\text{ kg} + 12.0\text{ kg}} = 0.526\text{ m/s}.$$

Note: We are displaying rounded intermediate values for practical purposes. However, the calculations are made using the unrounded values.

The spaceship's speed increase is given by the following.

$$v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$$

Solve for the mass ratio and substitute values to find the following.

$$\ln\left(\frac{M_i}{M_f}\right) = \frac{v_f - v_i}{v_e} = \frac{1.15 \times 10^3 \text{ m/s}}{2.25 \times 10^3 \text{ m/s}} = 0.511$$

$$e^{\ln(M_i/M_f)} = e^{0.511}$$

$$M_i/M_f = 1.67$$

Note: We are displaying rounded intermediate values for practical purposes. However, the calculations are made using the unrounded values.

An acceleration of $5g$ equals 49.0 m/s^2. By Newton's second law, this acceleration requires a net force, assumed equal to the thrust T, of the following magnitude.

$$F_{\text{net}} = T = ma = (3.50 \times 10^4 \text{ kg})(49.0 \text{ m/s}^2) = 1.72 \times 10^6 \text{ N}$$

A rocket engine's instantaneous thrust T is given by the following.

$$T = \left| v_e \frac{\Delta M}{\Delta t} \right|$$

Solve for the burn rate (change in mass per unit time) and substitute to find the following.

$$\left| \frac{\Delta M}{\Delta t} \right| = \frac{T}{v_e} = \frac{1.72 \times 10^6 \text{ N}}{2.70 \times 10^3 \text{ m/s}} = 635 \text{ kg/s}$$