Help: HW_7

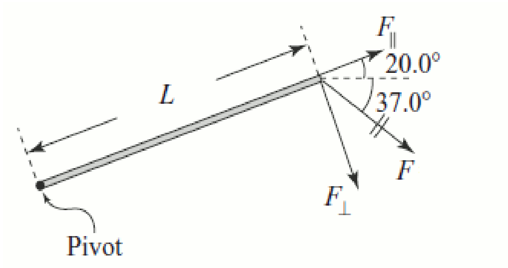
Solution or Explanation

Resolve the 112-N force into components parallel to and perpendicular to the rod, as

$F_{\parallel} = F\cos(20.0° + 37.0°) = F\cos 57.0°$

and

$F_{\perp} = F\sin(20.0° + 37.0°) = F\sin 57.0°$.



The lever arm of $F_{\perp}$ about the indicated pivot is 1.81 m, while that of $F_{||}$ is zero. The torque due to the 112-N force may be computed as the sum of the torques of its components, giving

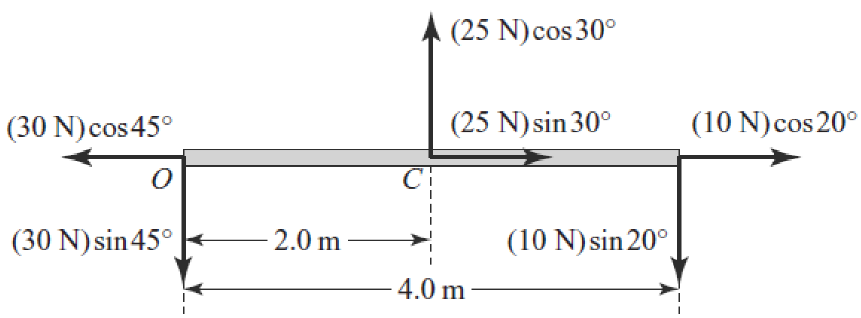
$\tau = F_{\parallel}(0) - F_{\perp}(1.81\text{ m}) = 0 - [(112\text{ N})\sin 57.0°](1.81\text{ m}) = -170\text{ N}\cdot\text{m}$

or

$\tau = 170\text{ N}\cdot\text{m}$ clockwise.

Solution or Explanation

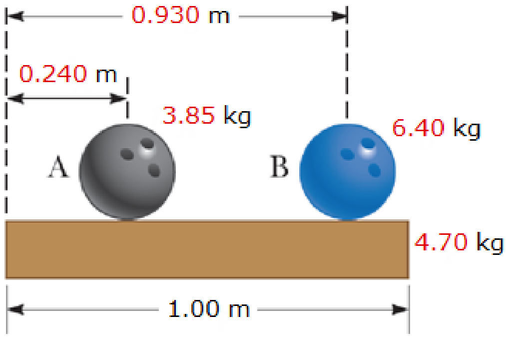
First resolve all of the forces shown in the figure below into components parallel to and perpendicular to the beam as shown in the sketch below.



(a) $\tau_O = +[(25\text{ N})\cos(30°)](2.0\text{ m}) - [(10\text{ N})\sin(20°)](4.0\text{ m}) = +30\text{ N}\cdot\text{m}$ or $\tau_O = 30\text{ N}\cdot\text{m}$ counterclockwise

(b) $\tau_C = +[(30\text{ N})\sin(45°)](2.0\text{ m}) - [(10\text{ N})\sin(20°)](2.0\text{ m}) = +36\text{ N}\cdot\text{m}$ or $\tau_C = 36\text{ N}\cdot\text{m}$ counterclockwise

HINT



0.617 m

Solution or Explanation

Both bowling balls and the plank are uniform so their centers of mass are located at their geometric centers. Take $x = 0$ at the left end of the plank. For the plank, $x_{p,\,cm} = \frac{1.00\text{ m}}{2} = 0.500\text{ m}$ and $m_p = 4.70\text{ kg}$. For the left bowling ball, $x_{A,\,cm} = 0.240\text{ m}$ and $m_A = 3.85\text{ kg}$. For the right bowling ball, $x_{B,\,cm} = 0.930\text{ m}$ and $m_B = 6.40\text{ kg}$. The x-component of the center of mass is then the following.

$$x_{cm} = \frac{\Sigma(m_i x_i)}{\Sigma m_i} = \frac{m_p x_{p,\,cm} + m_A x_{A,\,cm} + m_B x_{B,\,cm}}{m_p + m_A + m_B}$$

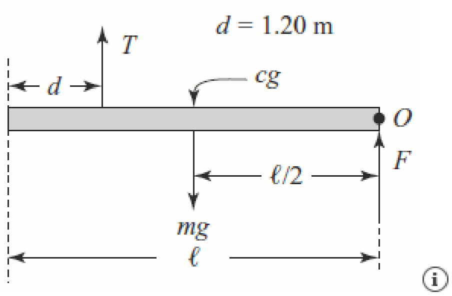
$$= \frac{(4.70\text{ kg})(0.500\text{ m}) + (3.85\text{ kg})(0.240\text{ m}) + (6.40\text{ kg})(0.930\text{ m})}{(4.70\text{ kg}) + (3.85\text{ kg}) + (6.40\text{ kg})}$$

$$= 0.617\text{ m}$$

Solution or Explanation

Note: We are displaying rounded intermediate values for practical purposes. However, the calculations are made using the unrounded values.

Since the beam is uniform, its center of gravity is at its geometric center.



Requiring that $\sum \tau = 0$ about an axis through point O and perpendicular to the page gives

$$F(0) + mg(\ell/2) - T(\ell - d) = 0.$$

(a) The tension in the rope must then be

$$T = \frac{mg(\ell/2)}{\ell - d} = \frac{(348\text{ N})(2.93\text{ m})}{5.85\text{ m} - 1.20\text{ m}} = 219\text{ N}.$$

(b) The force the column exerts is found from

$$\sum F_y = 0 \quad \Rightarrow \quad F + T - mg = 0$$

or

$$F = mg - T = 348\text{ N} - 219\text{ N} = 129\text{ N upward.}$$

A potter's wheel having a radius 0.49 m and a moment of inertia of 14.8 kg · m^2 is rotating freely at 51 rev/min. The potter can stop the wheel in 7.5 s by pressing a wet rag against the rim and exerting a radially inward force of 65 N. Find the effective coefficient of kinetic friction between the wheel and the wet rag.

0.331

Solution or Explanation

The angular acceleration is given below, with $\omega_f = 0$.

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{-\omega_i}{\Delta t}$$

Thus, the torque is the following.

$$\tau = I\alpha = -\left(\frac{I\omega_i}{\Delta t}\right)$$

But, the torque is also $\tau = -fr$, so the magnitude of the required friction force is calculated as follows.

$$f = \frac{I\omega_i}{r\Delta t} = \frac{(14.8\text{ kg}\cdot\text{m}^2)(51\text{ rev/min})}{(0.49\text{ m})(7.5\text{ s})}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 21.508\text{ N}$$

Therefore, we can compute the coefficient of kinetic friction.

$$\mu_k = \frac{f}{n} = \frac{21.508\text{ N}}{65\text{ N}} = 0.331$$

A model airplane with a mass of 0.745 kg is tethered by a wire so that it flies in a circle 29.2 m in radius. The airplane engine provides a net thrust of 0.792 N perpendicular to the tethering wire.

(a) Find the torque the net thrust produces about the center of the circle.

23.1 N · m

(b) Find the angular acceleration of the airplane when it is in level flight.

0.0364 rad/s^2

(c) Find the linear acceleration of the airplane tangent to its flight path.

1.06 m/s^2

Solution or Explanation

(a) $\tau = F \cdot r = (0.792\text{ N})(29.2\text{ m}) = 23.1\text{ N}\cdot\text{m}$

(b) $\alpha = \frac{\tau}{I} = \frac{\tau}{mr^2} = \frac{23.1\text{ N}\cdot\text{m}}{(0.745\text{ kg})(29.2\text{ m})^2} = 0.0364\text{ rad/s}^2$

(c) $a_t = r\alpha = (29.2\text{ m})(0.0364\text{ rad/s}^2) = 1.06\text{ m/s}^2$

Solution or Explanation

(a) The shell's translational kinetic energy is as follows.

$$KE_t = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(1.75 \text{ kg})(4.25 \text{ m/s})^2$$
$$= 15.8 \text{ J}$$

(b) An object's rotational kinetic energy is $KE_r = \frac{1}{2}I\omega^2$ where I is the moment of inertia about its center of mass and $\omega = v/R$ is its angular speed.

The moment of inertia for a thin spherical shell rotating about its center is $I = \frac{2}{3}MR^2$ so that the following is true.

$$KE_r = \frac{1}{2}I\omega^2$$
$$= \frac{1}{2}\left(\frac{2}{3}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{2}{3}\left(\frac{1}{2}Mv^2\right)$$
$$= \frac{2}{3}(KE_t) = \frac{2}{3}(15.8 \text{ J})$$
$$= 10.5 \text{ J}$$

A 270-N sphere 0.20 m in radius rolls without slipping 6.0 m down a ramp that is inclined at 34° with the horizontal. What is the angular speed of the sphere at the bottom of the slope if it starts from rest?

[ ] 34.3 rad/s

Solution or Explanation

Using conservation of mechanical energy,

$$(KE_t + KE_r + PE_g)_f = (KE_t + KE_r + PE_g)_i$$

or

$$\frac{1}{2}Mv_t^2 + \frac{1}{2}I\omega^2 + 0 = 0 + 0 + Mg(L \sin\theta).$$

Since $I = \frac{2}{5}MR^2$ for a solid sphere and $v_t = R\omega$ when rolling without slipping, this becomes

$$\frac{1}{2}MR^2\omega^2 + \frac{1}{5}MR^2\omega^2 = Mg(L \sin\theta)$$

and reduces to

$$\omega = \sqrt{\frac{10gL \sin\theta}{7R^2}} = \sqrt{\frac{10(9.8 \text{ m/s}^2)(6.0 \text{ m})\sin 34°}{7(0.20 \text{ m})^2}} = 34.3 \text{ rad/s}.$$

Solution or Explanation

(a) The rotational analog of Newton's second law in terms of angular momentum is $\Sigma\tau = \frac{\Delta L}{\Delta t}$. For a single torque τ acting on the hoop, the change in the hoop's angular momentum is the following.

$$\Delta L = \tau\,\Delta t = (1.15 \times 10^{-2}\ \text{N}\cdot\text{m})(1.80\ \text{s})$$
$$= 2.07 \times 10^{-2}\ \text{kg}\cdot\text{m}^2/\text{s}$$

(b) Solve for the change in angular speed using $\Delta L = I\Delta\omega$ with $I = MR^2$ (for the metal hoop) to find the following.

$$\Delta\omega = \frac{\Delta L}{MR^2} = \frac{2.07 \times 10^{-2}\ \text{kg}\cdot\text{m}^2/\text{s}}{(0.230\ \text{kg})(0.140\ \text{m})^2}$$
$$= 4.59\ \text{rad/s}$$

Solution or Explanation

Note: We are displaying rounded intermediate values for practical purposes. However, the calculations are made using the unrounded values.

The total moment of inertia of the system is

$$I_{\text{total}} = I_{\text{masses}} + I_{\text{student plus stool}} = 2(mr^2) + 3.0\ \text{kg}\cdot\text{m}^2.$$

Initially, $r = 1.0$ m, and $I_i = 2[(2.7\ \text{kg})(1.0\ \text{m})^2] + 3.0\ \text{kg}\cdot\text{m}^2 = 8.40\ \text{kg}\cdot\text{m}^2$.

Afterward, $r = 0.26$ m, so

$$I_f = 2[(2.7\ \text{kg})(0.26\ \text{m})^2] + 3.0\ \text{kg}\cdot\text{m}^2 = 3.37\ \text{kg}\cdot\text{m}^2.$$

(a) From conservation of angular momentum, $I_f\omega_f = I_i\omega_i$, or

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{8.40\ \text{kg}\cdot\text{m}^2}{3.37\ \text{kg}\cdot\text{m}^2}\right)(0.75\ \text{rad/s}) = 1.87\ \text{rad/s}$$

(b) $$KE_i = \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}(8.40\ \text{kg}\cdot\text{m}^2)(0.75\ \text{rad/s})^2 = 2.36\ \text{J}$$

$$KE_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(3.37\ \text{kg}\cdot\text{m}^2)(1.87\ \text{rad/s})^2 = 5.90\ \text{J}$$