Help – HW6

Solution or Explanation

(a) By convention, the counterclockwise direction is taken to be positive. Here, the tire is initially spinning clockwise at 3.80 rad/s so that ω_i = −3.80 rad/s. After 1.50 s, it has reversed direction to spin counterclockwise with ω_f = 3.80 rad/s. The tire's change in angular velocity is then the following.

$$\begin{aligned}\Delta\omega &= \omega_f - \omega_i \\ &= (3.80\text{ rad/s}) - (-3.80\text{ rad/s}) \\ &= 7.60\text{ rad/s}\end{aligned}$$

(b) Apply the definition of average angular acceleration to find the following.

$$\begin{aligned}\alpha_{\text{av}} &= \frac{\Delta\omega}{\Delta t} = \frac{7.60\text{ rad/s}}{1.50\text{ s}} \\ &= 5.07\text{ rad/s}^2\end{aligned}$$

(a) We have the following formula.

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

We use it to find the angular acceleration.

$$\alpha = \frac{0.20\text{ rev/s} - 0}{29.0\text{ s}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 0.0433\text{ rad/s}^2$$

(b) Yes. When an object starts from rest, its angular speed is related to the angular acceleration and time by the equation

$$\omega = \alpha(\Delta t).$$

Thus, the angular speed is directly proportional to both the angular acceleration and the time interval. It the time interval is held constant, doubling the angular acceleration will double the angular speed attained during the interval.

Solution or Explanation

(a) $\alpha = \dfrac{(2.15 \times 10^4 \text{ rev /min} - 0)}{3.30 \text{ s}} \left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\dfrac{1 \text{ min}}{60.0 \text{ s}}\right) = 682 \text{ rad/s}^2$

(b) $\theta = \overline{\omega}t = \left(\dfrac{\omega_f + \omega_0}{2}\right)t = \left[\dfrac{(2.15 \times 10^4 \text{ rev/min})(2\pi \text{ rad/1 rev})(1 \text{ min/60.0 s}) + 0}{2}\right](3.30 \text{ s}) = 3.71 \times 10^3 \text{ rad}$

The radius of the cylinder is $r = (2.39 \text{ mi})\left(\dfrac{1609 \text{ m}}{1 \text{ mi}}\right) = 3.85 \times 10^3$ m. Thus, from $a_c = r\omega^2$, the required angular velocity is

$$\omega = \sqrt{\frac{a_c}{r}} = \sqrt{\frac{9.80 \text{ m/s}^2}{3.85 \times 10^3 \text{ m}}} = 5.05 \times 10^{-2} \text{ rad/s.}$$

Note: We are displaying rounded intermediate values for practical purposes. However, the calculations are made using the unrounded values.

(a) Use the expression $a_c = \dfrac{v^2}{r} = r\omega^2$ to solve for the centrifuge's angular speed, ω.

$$\omega = \sqrt{\frac{a_c}{r}} = \sqrt{\frac{9g}{3.75 \text{ m}}}$$
$$= 4.85 \text{ rad/s}$$

(b) The linear speed of a person in the centrifuge is the following.

$$v_t = r\omega$$
$$= (3.75 \text{ m})(4.85 \text{ rad/s})$$
$$= 18.2 \text{ m/s}$$

Solution or Explanation

(a) $a_c = r\omega^2 = (3.00 \text{ m})(3.45 \text{ rad/s})^2 = 35.7 \text{ m/s}^2$

(b) $F_c = ma_c = (50.0 \text{ kg})(35.7 \text{ m/s}^2) = 1790 \text{ N}$

(c) We know the centripetal acceleration is produced by the force of friction. Therefore, the needed static friction force is $f_s = 1790$ N. Also, the normal force is $n = mg = 490$ N. Thus, the minimum coefficient of friction required is

$$\mu_s = \frac{(f_s)_{\text{max}}}{n} = \frac{1790 \text{ N}}{490 \text{ N}} = 3.64.$$

Such a large coefficient of friction is unrealistic, and she will not be able to stay on the merry-go-round.

Solution or Explanation

(a) Since the 1.1 kg mass is in equilibrium, the tension in the string must be equal to the force of gravity acting on the 1.1 kg mass.

$$T = mg = (1.1 \text{ kg})(9.8 \text{ m/s}^2) = 10.8 \text{ N}$$

(b) The tension in the string must cause the centripetal acceleration of the puck.

$$F_c = T = 10.8 \text{ N}$$

(c) We know the following relationship between the centripetal force of the puck and its tangential velocity.

$$F_c = m_{\text{puck}}\left(\frac{v_t^2}{r}\right)$$

We solve for tangential velocity in terms of centripetal force and mass.

$$F_c = m_{\text{puck}}\left(\frac{v_t^2}{r}\right) \Rightarrow \frac{F_c \cdot r}{m_{\text{puck}}} = v_t^2 \Rightarrow v_t = \sqrt{\frac{r \cdot F_c}{m_{\text{puck}}}} = \sqrt{\frac{(1.0 \text{ m})(10.8 \text{ N})}{(0.24 \text{ kg})}} = 6.70 \text{ m/s}$$

Solution or Explanation

From Kepler's third law, written in the form suitable for bodies orbiting Mars, we have $T^2 = (4\pi^2/GM_{\text{Mars}})r^3$, so the mass of Mars, computed from the given data, must be

$$M_{\text{Mars}} = \left(\frac{4\pi^2}{GT^2}\right)r^3 = \left(\frac{4\pi^2}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(2.8 \times 10^4 \text{ s})^2}\right)(9.4 \times 10^6 \text{ m})^3 = 6.27 \times 10^{23} \text{ kg}.$$

Solution or Explanation

(a) To find the gravitational force exerted by Earth on the space station, substitute values into Newton's law of universal gravitation.

$$F = G\frac{m_1 m_2}{r^2} = G\frac{M_E m_{ss}}{r_{ss}^{2}}$$
$$= (6.67 \times 10^{-11}\ (\text{N} \cdot \text{m}^2)/\text{kg}^2)\frac{(5.98 \times 10^{24}\ \text{kg})(4.19 \times 10^{5}\ \text{kg})}{(6.79 \times 10^{6}\ \text{m})^2}$$
$$= 3.63 \times 10^{6}\ \text{N}$$

(b) The gravitational potential energy of the Earth-space station system is the following.

$$PE = -G\frac{M_E m_{ss}}{r_{ss}} = -(6.67 \times 10^{-11}\ (\text{N} \cdot \text{m}^2)/\text{kg}^2)\frac{(5.98 \times 10^{24}\ \text{kg})(4.19 \times 10^{5}\ \text{kg})}{6.79 \times 10^{6}\ \text{m}}$$
$$= -2.46 \times 10^{13}\ \text{J}$$

(c) The astronaut's weight is $w = mg$ where $g = \frac{GM_E}{r_{ss}^{2}}$. Substitute values to find the following.

$$w = mg = m\left(\frac{GM_E}{r_{ss}^{2}}\right)$$
$$= (79.9\ \text{kg})\left(\frac{(6.67 \times 10^{-11}\ (\text{N} \cdot \text{m}^2)/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{(6.79 \times 10^{6}\ \text{m})^2}\right)$$
$$= 692\ \text{N}$$

Solution or Explanation

The radius of the satellite's orbit is

$$r = R_E + h = 6.38 \times 10^{6}\ \text{m} + (6.16 \times 10^{2}\ \text{mi})(1{,}609\ \text{m}/1\ \text{mi}) = 7.37 \times 10^{6}\ \text{m}$$

(a) The required centripetal acceleration is produced by the gravitational force, so $m(v_t^2/r) = GM_E m/r^2$, which gives $v_t = \sqrt{GM_E/r^2}$.

$$v_t = \sqrt{\left(6.67 \times 10^{-11}\frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\frac{(5.98 \times 10^{24}\ \text{kg})}{7.37 \times 10^{6}\ \text{m}}} = 7.36 \times 10^{3}\ \text{m/s}$$

(b) The time for one complete revolution is

$$T = \frac{2\pi r}{v_t} = \frac{2\pi(7.37 \times 10^{6}\ \text{m})}{7.36 \times 10^{3}\ \text{m/s}} = 6.30 \times 10^{3}\ \text{s} = 105\ \text{min}$$