Help: HW_4

A cable exerts a constant upward tension of magnitude 1.82×10^4 N on a 1.60×10^3 kg elevator as it rises through a vertical distance of 1.50 m.

HINT

(a) Find the work done by the tension force on the elevator (in J).

[ ] 27300 J

(b) Find the work done by the force of gravity on the elevator (in J).

[ ] -23500 J

Solution or Explanation

(a) The work done by tension T on the elevator is $W_T = (T \cos(\theta))\, d$ where $T = 1.82 \times 10^4$ N is the tension force, $d = 1.50$ m is the vertical displacement, and θ is the angle between the tension and the displacement. Here, both the tension and the displacement are directed vertically upward so that $\theta = 0°$ and the work done is given by the following.

$$W_T = (T \cos(\theta))d = ((1.82 \times 10^4 \text{ N}) \cos(0°))(1.50 \text{ m}) = 2.73 \times 10^4 \text{ J}$$

(b) The work done by the force of gravity on the elevator is $W_g = (mg \cos(\theta))\, d$ where $m = 1.60 \times 10^3$ kg is the elevator's mass, $d = 1.50$ m is the vertical displacement, and θ is the angle between the gravity force and the displacement. The downward gravity force and the upward displacement are in opposite directions so that $\theta = 180°$ and the work done is given by the following.

$$W_g = (mg \cos(\theta))\, d = ((1.60 \times 10^3 \text{ kg})(9.80 \text{ m/s}^2) \cos(180°))(1.50 \text{ m}) = -2.35 \times 10^4 \text{ J}$$

A horizontal force of 150 N is used to push a 39.5-kg packing crate a distance of 5.75 m on a rough horizontal surface. If the crate moves at constant speed, find each of the following.

(a) the work done by the 150-N force

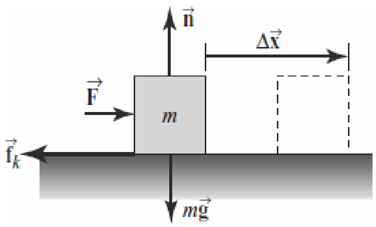
863 J

(b) the coefficient of kinetic friction between the crate and the surface

0.387

Solution or Explanation

(a) $W_F = F(\Delta x)\cos\theta = (150\text{ N})(5.75\text{ m})\cos 0° = 863\text{ J}$



(b) Since the crate moves at constant velocity, $a_x = a_y = 0$. Thus,

$$\Sigma F_x = 0 \Rightarrow f_k = F = 150\text{ N}.$$

Also,

$$\Sigma F_y = 0 \Rightarrow n = mg = (39.5\text{ kg})(9.80\text{ m/s}^2) = 387\text{ N},$$

so

$$\mu_k = \frac{f_k}{n} = \frac{150\text{ N}}{387\text{ N}} = 0.387.$$

A mechanic pushes a 2.40×10^3-kg car from rest to a speed of v, doing 4,560 J of work in the process. During this time, the car moves 20.0 m. Neglecting friction between car and road, find v and the horizontal force exerted on the car.

(a) the speed v

1.95 m/s

(b) the horizontal force exerted on the car (Enter the magnitude.)

228 N

Solution or Explanation

(a) The work-energy theorem, $W_{net} = KE_f - KE_i$, gives

$$4560 \text{ J} = \frac{1}{2}(2.40 \times 10^3 \text{ kg})v^2 - 0, \text{ or } v = 1.95 \text{ m/s.}$$

(b) $W = (F \cos \theta)s = (F \cos 0°)(20.0 \text{ m}) = 4560 \text{ J}$, so $F = 228$ N.

A 60.0 kg cheetah accelerates from rest to its top speed of 31.3 m/s.

HINT

(a) How much net work (in J) is required for the cheetah to reach its top speed?

[ ] 29400 J

(b) One food Calorie equals 4186 J. How many Calories of net work are required for the cheetah to reach its top speed? Note: Due to inefficiencies in converting chemical energy to mechanical energy, the amount calculated here is only a fraction of the energy that must be produced by the cheetah's body.

[ ] 7.02 Cal

Solution or Explanation

Note: We are displaying rounded intermediate values for practical purposes. However, the calculations are made using the unrounded values.

(a) From the work-energy theorem, the required net work equals the change in the cheetah's kinetic energy. The cheetah starts from rest, so its initial kinetic energy, KE_i, equals zero and we have the following.

$$\begin{aligned} W_{net} &= \Delta KE = KE_f - KE_i \\ &= KE_f \\ &= \frac{1}{2}mv^2 = \frac{1}{2}(60.0\text{ kg})(31.3\text{ m/s})^2 \\ &= 2.94 \times 10^4\text{ J} \end{aligned}$$

(b) Converting to Calories gives the following.

$$W_{net} = (2.94 \times 10^4\text{ J})\left(\frac{1\text{ Calorie}}{4186\text{ J}}\right) = 7.02\text{ Calories}$$

When a 2.30-kg object is hung vertically on a certain light spring described by Hooke's law, the spring stretches 2.20 cm.

(a) What is the force constant of the spring?

[ ] 1020 N/m

(b) If the 2.30-kg object is removed, how far will the spring stretch if a 1.15-kg block is hung on it?

[ ] 1.1 cm

(c) How much work must an external agent do to stretch the same spring 9.00 cm from its unstretched position?

[ ] 4.15 J

Solution or Explanation

(a) The force stretching the spring is the weight of the suspended object. Therefore, the force constant of the spring is

$$k = \left|\frac{F_g}{\Delta x}\right| = \frac{mg}{|\Delta x|} = \frac{(2.30\text{ kg})(9.80\text{ m/s}^2)}{2.20 \times 10^{-2}\text{ m}} = 1.02 \times 10^3\text{ N/m.}$$

(b) If a 1.15 kg block replaces the original 2.30 kg suspended object, the force applied to the spring (weight of the suspended object) will be one-half the original stretching force. Since, for a spring obeying Hooke's law, the elongation is directly proportional to the stretching force, the amount the spring stretches now is

$$(\Delta x)_2 = \frac{1}{2}(\Delta x)_1 = \frac{1}{2}(2.20\text{ cm}) = 1.10\text{ cm.}$$

(c) The work an external agent must do *on* the initially unstretched spring to produce an elongation x_f is equal to the potential energy stored in the spring at the elongation

$$W_{\text{done on spring}} = (PE_s)_f - (PE_s)_i = \frac{1}{2}kx_f^2 - 0 = \frac{1}{2}(1.02 \times 10^3\text{ N/m})(9.00 \times 10^{-2}\text{ m})^2 = 4.15\text{ J.}$$

A 48-kg pole vaulter running at 10 m/s vaults over the bar. Her speed when she is above the bar is 1.1 m/s. Neglect air resistance, as well as any energy absorbed by the pole, and determine her altitude as she crosses the bar.

[ ] 5.04 m

Solution or Explanation

Using conservation of mechanical energy, we have

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + 0$$

or

$$y_f = \frac{v_i^2 - v_f^2}{2g} = \frac{(10\text{ m/s})^2 - (1.1\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 5.04\text{ m}.$$

A 31.0-kg child on a 4.00-m-long swing is released from rest when the ropes of the swing make an angle of 34.0° with the vertical.

(a) Neglecting friction, find the child's speed at the lowest position.

3.66 m/s

(b) If the actual speed of the child at the lowest position is 3.20 m/s, what is the mechanical energy lost due to friction?

49 J

Solution or Explanation

(a) Choose $PE_g = 0$ at the level of the bottom of the arc. The child's initial vertical displacement from this level is

$$y_i = (4.00\text{ m})(1 - \cos 34.0°) = 0.684\text{ m}.$$

In the absence of friction, we use conservation of mechanical energy as

$$(KE + PE_g)_f = (KE + PE_g)_i, \text{ or } \frac{1}{2}mv_f^2 + 0 = 0 + mgy_i,$$

which gives

$$v_f = \sqrt{2gy_i} = \sqrt{2(9.80\text{ m/s}^2)(0.684\text{ m})} = 3.66\text{ m/s}.$$

(b) With a nonconservative force present, we use

$$W_{nc} = (KE + PE_g)_f - (KE + PE_g)_i = \left(\frac{1}{2}mv_f^2 + 0\right) - (0 + mgy_i),$$

or

$$W_{nc} = m\left(\frac{v_f^2}{2} - gy_i\right)$$

$$= (31.0\text{ kg})\left[\frac{(3.20\text{ m/s})^2}{2} - (9.80\text{ m/s}^2)(0.684\text{ m})\right] = -49.0\text{ J}.$$

Thus, 49.0 J of energy is spent overcoming friction.

A model train powered by an electric motor accelerates from rest to 0.520 m/s in 17.0 ms. The total mass of the train is 585 g. What is the average power (in W) delivered to the train by the motor during its acceleration?

[ ] 4.65 W

Solution or Explanation

Assuming a level track, $PE_f = PE_i$, and the work done on the train is

$$W_{nc} = (KE + PE)_f - (KE + PE)_i$$
$$= \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}(0.585 \text{ kg})[(0.520 \text{ m/s})^2 - 0] = 0.0791 \text{ J}.$$

The power delivered by the motor is then

$$P = \frac{W_{nc}}{\Delta t} = \frac{0.0791 \text{ J}}{1.70 \times 10^{-2} \text{ s}} = 4.65 \text{ W}.$$

Under normal conditions the human heart converts about 12.7 J of chemical energy per second into 1.27 W of mechanical power as it pumps blood throughout the body.

HINT

(a) Determine the number of Calories required to power the heart for one day, given that 1 Calorie equals 4186 J.

[ ] 262 Cal

(b) Metabolizing 1 kg of fat can release about 9000 Calories of energy. What mass of metabolized fat (in kg) would power the heart for one day?

[ ] 0.0291 kg

Solution or Explanation

Note: We are displaying rounded intermediate values for practical purposes. However, the calculations are made using the unrounded values.

(a) There are 8.64×10^4 s in a day. The heart consumes 12.7 J each second so its daily energy requirement is the following.

$$E = \left(\frac{12.7\text{ J}}{1\text{ s}}\right)\left(\frac{8.64 \times 10^4\text{ s}}{1\text{ day}}\right)\left(1\text{ day}\right) = 1.10 \times 10^6\text{ J}$$

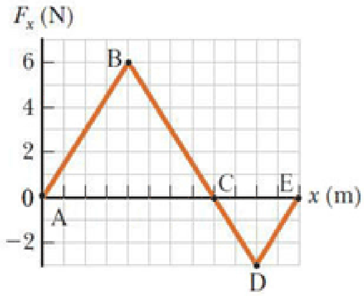
Using the conversion factor 1 Calorie = 4186 J, the daily energy requirement in Calories is the following.

$$E = (1.10 \times 10^6\text{ J})\left(\frac{1\text{ Calorie}}{4186\text{ J}}\right) = 262\text{ Calories}$$

(b) Metabolizing 1 kg of fat can release about 9000 Calories. The mass of fat required to provide one day's worth of Calories for the heart is the following.

$$m = \frac{E}{9000\text{ Calories/kg}} = \frac{262\text{ Calories}}{9000\text{ Calories/kg}}$$
$$= 2.91 \times 10^{-2}\text{ kg}$$

The force acting on a particle varies as in the figure below. (The x axis is marked in increments of 2.00 m.)



Find the work done by the force as the particle moves across the following distances.

(a) from x = 0 m to x = 16.0 m

48 J

(b) from x = 16.0 m to x = 24.0 m

-12 J

(c) from x = 0 m to x = 24.0 m

36 J