Solution or Explanation

(a) After moving half way around the circle of radius $R = 3.10 \times 10^3$ m, the magnitude of the airplane's displacement $\Delta\vec{\mathbf{r}}$ is equal to the circle's diameter,

$$\Delta r = 2R = 6.20 \times 10^3 \text{ m}.$$

(b) The magnitude of the airplane's average velocity is the following.

$$v_{av} = \frac{\Delta r}{\Delta t} = \frac{6.20 \times 10^3 \text{ m}}{136 \text{ s}}$$
$$= 45.6 \text{ m/s}$$

(c) The airplane's average speed v equals the path length divided by the elapsed time. After moving half way around the circle, the plane has traveled over a path length equal to half the circle's circumference. Its average speed is the following.

$$\text{average speed} = \frac{\frac{1}{2}(2\pi R)}{\Delta t} = \frac{\pi(3.10 \times 10^3 \text{ m})}{136 \text{ s}}$$
$$= 71.6 \text{ m/s}$$

(a) The components of the ant's displacement are the following.

$$\Delta x = x_f - x_i = 0.350 \text{ m} - 0.100 \text{ m} = 0.250 \text{ m}$$
$$\Delta y = y_f - y_i = 0.300 \text{ m} - 0.150 \text{ m} = 0.150 \text{ m}$$

(b) The average velocity has the following components.

$$v_{\text{av}, x} = \frac{\Delta x}{\Delta t} = \frac{0.250 \text{ m}}{4.60 \text{ s}} = 0.0543 \text{ m/s}$$
$$v_{\text{av}, y} = \frac{\Delta y}{\Delta t} = \frac{0.150 \text{ m}}{4.60 \text{ s}} = 0.0326 \text{ m/s}$$

(c) The average acceleration has the following components.

$$a_{\text{av}, x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{x, f} - v_{x, i}}{\Delta t} = \frac{0.190 \text{ m/s} - 0}{4.60 \text{ s}} = 0.0413 \text{ m/s}^2$$
$$a_{\text{av}, y} = \frac{\Delta v_y}{\Delta t} = \frac{v_{y, f} - v_{y, i}}{\Delta t} = \frac{0 - 0.190 \text{ m/s}}{4.60 \text{ s}} = -0.0413 \text{ m/s}^2$$

A rabbit is moving in the positive x-direction at 1.90 m/s when it spots a predator and accelerates to a velocity of 12.1 m/s along the positive y-axis, all in 1.20 s. Determine the x-component and the y-component of the rabbit's acceleration. (Enter your answers in m/s^2. Indicate the direction with the signs of your answers.)

Solution or Explanation

(a) The x-component of the rabbit's average acceleration is the following.

$$a_{av,x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{x,f} - v_{x,i}}{\Delta t} = \frac{(0\text{ m/s}) - (1.90\text{ m/s})}{1.20\text{ s}} = -1.58\text{ m/s}^2$$

(b) The y-component of the rabbit's average acceleration is the following.

$$a_{av,y} = \frac{\Delta v_y}{\Delta t} = \frac{v_{y,f} - v_{y,i}}{\Delta t} = \frac{(-13.9\text{ m/s}) - (0\text{ m/s})}{1.20\text{ s}} = -11.6\text{ m/s}^2$$

One of the fastest recorded pitches in major-league baseball, thrown by Tim Lincecum in 2009, was clocked at 101.0 mi/h. If a pitch were thrown horizontally with this velocity, how far would the ball fall vertically by the time it reached home plate, 60.5 ft away?

Solution or Explanation

$v_{0x} = (101 \text{ mi/h})\left(\frac{0.447 \text{ m/s}}{1 \text{ mi/h}}\right) = 45.1 \text{ m/s}$ and $\Delta x = (60.5 \text{ ft})\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 18.4 \text{ m}$

The time to reach home plate is $t = \frac{\Delta x}{v_{0x}} = \frac{18.4 \text{ m}}{45.1 \text{ m/s}} = 0.408 \text{ s}$.

In this time interval, the vertical displacement is

$$\Delta y = v_{oy}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.408 \text{ s})^2 = -0.817 \text{ m}.$$

Thus, the ball drops vertically $(0.817 \text{ m})\left((3.281 \text{ ft})/(1 \text{ m})\right) = 2.68 \text{ ft}$.

Solution or Explanation

We choose our origin at the initial position of the projectile. After 3.1 s, it is at ground level, so the vertical displacement is $\Delta y = -H$.

To find H, we use the following equation.

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$$

We then plug in the information we have to find H.

$$-H = (13\ \text{m/s})(\sin 30°)(3.1\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(3.1\ \text{s})^2 \text{ or } H = \underline{26.9\ \text{m}}$$

An artillery shell is fired with an initial velocity of 300 m/s at 53.5° above the horizontal. To clear an avalanche, it explodes on a mountainside 35.5 s after firing. What are the *x*- and *y*-coordinates of the shell where it explodes, relative to its firing point?

$x =$ [ ] 6330 m

$y =$ [ ] 2390 m

Solution or Explanation

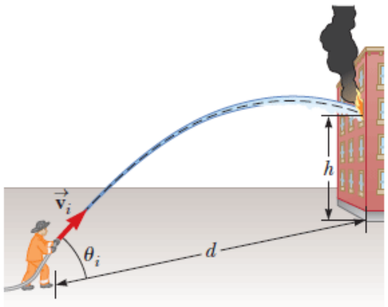
The horizontal displacement at t = 35.5 s is calculated as follows.

$$x = v_{0t}t = (v_o \cos\theta)t = (300\text{ m/s})(\cos 53.5°)(35.5\text{ s}) = \underline{6330\text{ m}}$$

The vertical displacement at t = 35.5 s is calculated as follows.

$$y = v_{0y}t + \frac{1}{2}a_y t^2 = (v_0 \sin\theta)t - \frac{1}{2}gt^2 y = (300\text{ m/s})(\sin 53.5°)(35.5\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(35.5\text{ s})^2 = \underline{2390\text{ m}}$$

Water from a fire hose is directed toward a building as shown in the figure below. The water leaves the hose at a speed of v_i = 40.0 m/s and at an angle of θ_i = 42.0° above the horizontal. The base of the hose (at ground level) is a horizontal distance d = 56.0 m away from the building. Find the height h (in m) where the water strikes the building.



33 m

Solution or Explanation

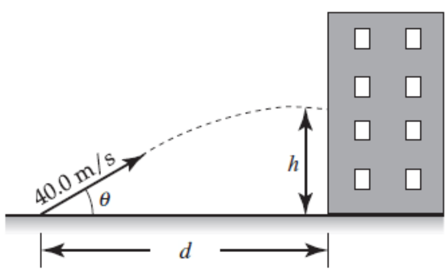
Note: We are displaying rounded intermediate values for practical purposes. However, the calculations are made using the unrounded values.

The components of the initial velocity are

$$v_{0x} = (40.0 \text{ m/s})\cos(42.0°) = 29.7 \text{ m/s}$$

and

$$v_{0y} = (40.0 \text{ m/s})\sin(42.0°) = 26.8 \text{ m/s}.$$



The time for the water to reach the building is

$$t = \frac{\Delta x}{v_{0x}} = \frac{56.0 \text{ m}}{29.7 \text{ m/s}} = 1.88 \text{ s}.$$

The height of the water at this time is $h = \Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, or

$$h = (26.8 \text{ m/s})(1.88 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(1.88 \text{ s})^2 = 33.0 \text{ m}.$$

The best leaper in the animal kingdom is the puma, which can jump to a height of 3.7 m when leaving the ground at an angle of 45°. With what speed must the animal leave the ground to reach that height?

[ ] 12 m/s

Solution or Explanation

At the maximum height $v_y = 0$, and the time to reach this height is found from

$$v_y = v_{0y} + a_y t \text{ as } t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - v_{0y}}{-g} = \frac{v_{0y}}{g}.$$

The vertical displacement that has occurred during this time is

$$(\Delta y)_{max} = (v_y)_{av} t = \left(\frac{v_y + v_{0y}}{2}\right)t = \left(\frac{0 + v_{0y}}{2}\right)\left(\frac{v_{0y}}{g}\right) = \frac{v_{0y}^2}{2g}.$$

If $(\Delta y)_{max} = 3.7$ m, we find

$$v_{0y} = \sqrt{2g(\Delta y)_{max}} = \sqrt{2(9.80 \text{ m/s}^2)(3.7 \text{ m})} = 8.5 \text{ m/s}$$

and if the angle of projection is $\theta = 45°$, the launch speed is

$$v_0 = \frac{v_{0y}}{\sin(\theta)} = \frac{8.5 \text{ m/s}}{\sin(45°)} = 12 \text{ m/s}.$$

Suppose a boat moves at 12.6 m/s relative to the water. If the boat is in a river with the current directed east at 3.30 m/s, what is the boat's speed relative to the ground when it is heading east, with the current, and west, against the current? (Enter your answers in m/s.)

HINT

(a) east with the current

[ ] 15.9 m/s

(b) west against the current

[ ] 9.3 m/s

Solution or Explanation

Let east be the positive direction so that the water's velocity relative to the ground is v_{WG} = +3.30 m/s. When the boat is moving east (with the current), its velocity relative to the water is v_{BW} = +12.6 m/s. When it is moving west (against the current), its velocity relative to the water is v_{BW} = −12.6 m/s.

(a) The boat's velocity relative to the ground when heading east is the following.

$$v_{BG} = v_{BW} + v_{WG} = +12.6 \text{ m/s} + 3.30 \text{ m/s} = 15.9 \text{ m/s}$$

Its ground speed is therefore $|v_{BG}|$ = 15.9 m/s.

(b) The boat's velocity relative to the ground when heading west is the following.

$$v_{BG} = v_{BW} + v_{WG} = -12.6 \text{ m/s} + 3.30 \text{ m/s} = -9.30 \text{ m/s}$$

Its ground speed is therefore $|v_{BG}|$ = 9.30 m/s.

Suppose a boat moves at 20.0 m/s relative to the water. If the boat is in a river with the current directed east at 1.90 m/s, what is the boat's speed relative to the ground when it is heading east, with the current, and west, against the current? (Enter your answers in m/s.)

HINT

(a) east with the current

[ ] 21.9 m/s

(b) west against the current

[ ] 18.1 m/s

Solution or Explanation

Let east be the positive direction so that the water's velocity relative to the ground is v_{WG} = +1.90 m/s. When the boat is moving east (with the current), its velocity relative to the water is v_{BW} = +20.0 m/s. When it is moving west (against the current), its velocity relative to the water is v_{BW} = −20.0 m/s.

(a) The boat's velocity relative to the ground when heading east is the following.

$$v_{BG} = v_{BW} + v_{WG} = +20.0 \text{ m/s} + 1.90 \text{ m/s} = 21.9 \text{ m/s}$$

Its ground speed is therefore $|v_{BG}|$ = 21.9 m/s.

(b) The boat's velocity relative to the ground when heading west is the following.

$$v_{BG} = v_{BW} + v_{WG} = -20.0 \text{ m/s} + 1.90 \text{ m/s} = -18.1 \text{ m/s}$$

Its ground speed is therefore $|v_{BG}|$ = 18.1 m/s.

An airplane maintains a speed of 585 km/h relative to the air it is flying through as it makes a trip to a city 675 km away to the north. (Assume north is the positive y-direction and east is the positive x-direction.)

(a) What time interval is required for the trip if the plane flies through a headwind blowing at 36.3 km/h toward the south?

1.23 h

(b) What time interval is required if there is a tailwind with the same speed?

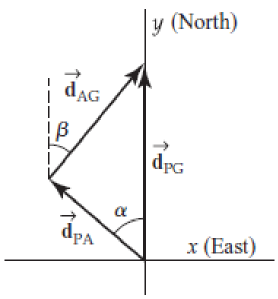
1.09 h

(c) What time interval is required if there is a crosswind blowing at 36.3 km/h to the east relative to the ground?

1.16 h

Solution or Explanation

During the trip of duration t, the displacement of the plane relative to the ground, $\vec{\mathbf{d}}_{PG}$, is to have magnitude of 675 km and be directed due north. We choose the positive y-axis to be directed northward and the positive x-axis directed eastward. During the trip, the plane's displacement relative to the air $\vec{\mathbf{d}}_{PA}$ has magnitude $|\vec{\mathbf{d}}_{PA}|$ = (585 km/h)t and is directed at some angle α relative to the y-axis. The displacement of the air relative to the ground, $\vec{\mathbf{d}}_{AG}$ has magnitude $|\vec{\mathbf{d}}_{AG}|$ = (36.3 km/h)t and is assumed to be at some angle β from the y-axis.



Since these relative displacements are related by $\vec{\mathbf{d}}_{PA} = \vec{\mathbf{d}}_{PG} - \vec{\mathbf{d}}_{AG}$, or $\vec{\mathbf{d}}_{PG} = \vec{\mathbf{d}}_{PA} + \vec{\mathbf{d}}_{AG}$, they form a vector triangle as shown above. Equating the x-components in the vector triangle gives

$$-|\vec{\mathbf{d}}_{PA}| \sin\alpha + |\vec{\mathbf{d}}_{AG}| \sin\beta = 0,$$

or

$$-(585 \text{ km/h})t \sin\alpha + (36.3 \text{ km/h})t \sin\beta = 0 \text{ and } \sin\alpha = \left(\frac{36.3 \text{ km/h}}{585 \text{ km/h}}\right)\sin\beta \qquad \textbf{[1]}$$

Equating y-components in the vector triangle gives

$$|\vec{\mathbf{d}}_{PA}| \cos\alpha + |\vec{\mathbf{d}}_{AG}| \cos\beta = |\vec{\mathbf{d}}_{PG}|, \text{ or } (585 \text{ km/h})t \cos\alpha + (36.3 \text{ km/h})t \cos\beta = 675 \text{ km} \qquad \textbf{[2]}$$

(a) The wind blows toward south ($\beta = 180°$) and is a headwind for the plane ($\alpha = 0°$). Then, Equation [2] gives

$$[585 \text{ km/h} - 36.3 \text{ km/h}]t = 675 \text{ km or } t = \frac{675 \text{ km}}{549 \text{ km/h}} = 1.23 \text{ h}$$

(b) The wind blows northward as a tailwind ($\alpha = \beta = 0°$), and Equation [2] yields

$$[585 \text{ km/h} + 36.3 \text{ km/h}]t = 675 \text{ km or } t = \frac{675 \text{ km}}{621 \text{ km/h}} = 1.09 \text{ h}$$

(c) The blows due east, so $\beta = 90.0°$. Then Equation [1] requires that

$$\sin\alpha = \left(\frac{36.3 \text{ km/h}}{585 \text{ km/h}}\right)\sin 90.0° = 0.0621 \text{ and } \alpha = 3.56°$$

or the plane must fly 3.56° W of N relative to the air to maintain a due north heading relative to the ground.
Finally, Equation [2] for this case gives

$$[(585 \text{ km/h})\cos 3.56° + (36.3 \text{ km/h})\cos 90.0°]t = 675 \text{ km or } t = \frac{675 \text{ km}}{(584 \text{ km/h} + 0)} = 1.16 \text{ h}$$