Mathematics: Physics is a quantitative science that requires mathematics in fundamental ways. We will use math freely and it is assumed that you have a thorough understanding of algebra and trigonometry. We will also use some calculus over the course of the semester, although no prior knowledge is assumed. You should have a calculator for this course and I will assume you

know how to use it

- Algebra
  - Linear equations:
    - $a x + b = 0 \Rightarrow x = -b/a$
  - Second order equations:

$$ax^2 + bx + c = 0 \qquad x_{1,2} = (-b \pm \sqrt{b^2 - 4ac})/(2a)$$

  - Exponential and logarithms:

$$\exp(-ax) = b \Rightarrow x = -\ln(b)/a$$

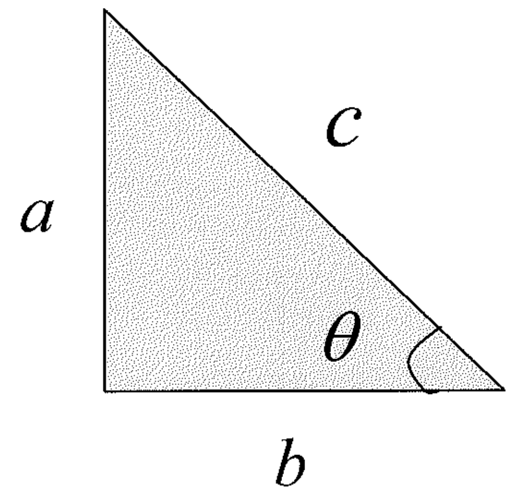
MATH

- Trigonometry

$$\sin\theta = \frac{a}{c} \qquad \cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{a}{b} \qquad \cot\theta = \frac{b}{a}$$

$$a^2 + b^2 = c^2$$



MATH

- Geometry
  - Perimeter and Area of Circle

    $$P_{circ} = 2\pi R \quad A_{circ} = \pi R^2$$

  - Area of a Sphere

    $$A_{Sph} = 4\pi R^2$$

  - Volume of a Sphere

    $$V_{Sph} = \frac{4}{3}\pi R^3$$

Dimensional Analysis

- Fundamental Units

  L - Length M - Mass

  T - Time θ - Temperature

- All other units can be expressed in terms of fundamental units. For example,

  V - Velocity F - Force E - Energy

$$V = \frac{L}{T} \qquad F = \frac{ML}{T^2} \qquad E = \frac{ML^2}{T^2} = FL$$

27. SOLVE We need to convert mi to m and h to s:

$$v(\text{cheetah}) = \left(70\frac{\text{mi}}{\text{h}}\right)\times\left(\frac{\text{h}}{3600\ \text{s}}\right)\times\left(\frac{1609\ \text{m}}{\text{mi}}\right) = 31.3\frac{\text{m}}{\text{s}}$$

Below is a list with the next 4 fastest animals:

Pronghorn Antelope 61 mph (98 km per hour)
Wildebeest 50 mph (80 km per hour)
Lion 50 mph (80 km per hour)
Thomson's Gazelle 50 mph (80 km per hour)

Table 1.4

TABLE 1.4 Some SI Prefixes*

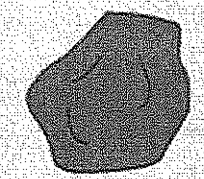
Power of 10	Prefix	Abbreviation
10^{-18}	Atto	a
10^{-15}	Femto	f
10^{-12}	Pico	p
10^{-9}	Nano	n
10^{-6}	Micro	μ
10^{-3}	Milli	m
10^{-2}	Centi	c
10^{3}	Kilo	k
10^{6}	Mega	M
10^{9}	Giga	G
10^{12}	Tera	T
10^{15}	Peta	P
10^{18}	Exa	E

*For a more complete list, see Appendix B.

5cc

A Comment on Significant Digits

No measurements are exact, and it is important to state experimental results with a number of significant digits which give a reasonable impression of the accuracy of the measurement. Consider the following example of a measured density.



Suppose you measured the mass of a rock to be 8.2 grams and its volume to be 2.3 cm^3. How accurately can you determine its density?

Mass measured to 2 significant digits

What's wrong with this calculation?

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{8.2\ \text{gm}}{2.3\ \text{cm}^3} = 3.56\underbrace{521739}_{\text{Not science, but fiction!}}\ \frac{\text{gm}}{\text{cm}^3}$$

Volume measured to 2 significant digits

Maybe this doesn't look as impressive, but it is a better answer.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{8.2\ \text{gm}}{2.3\ \text{cm}^3} = 3.57\ \frac{\text{gm}}{\text{cm}^3}$$

No fictional digits - - well, maybe the 7 is stretching it. It is reasonable to carry an extra digit or two in intermediate calculations.

$$\text{Density} = 3.6\ \frac{\text{gm}}{\text{cm}^3}$$

It is better to keep only the number of digits of your weakest measurement for your final answer.

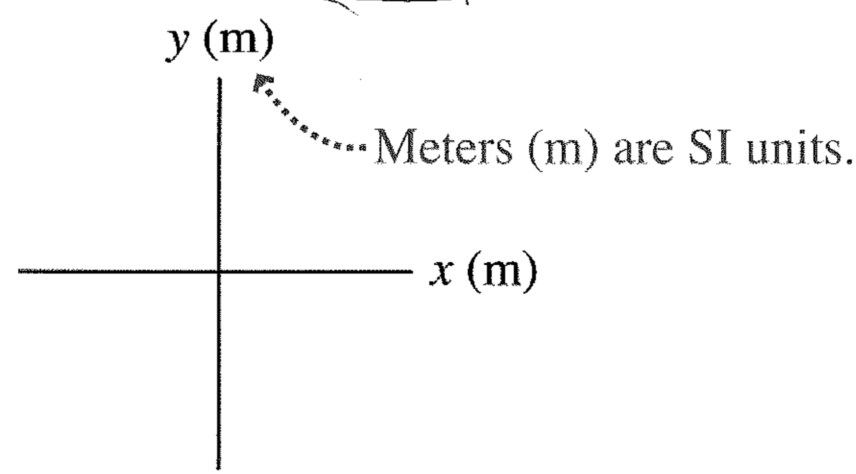
(digits)

The number of significant figures in 0.00150 is:

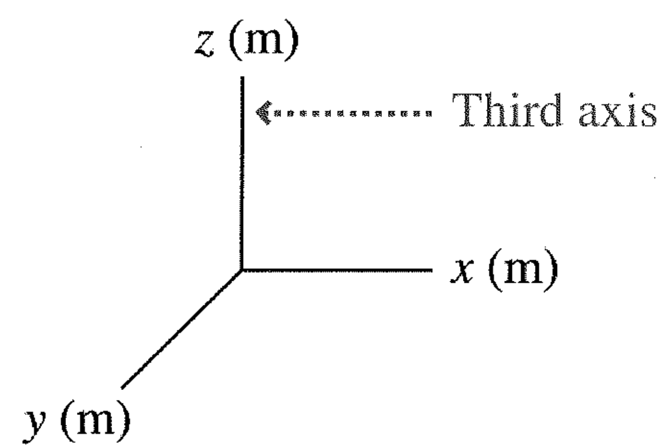
1) 2
2) 3
3) 4
4) 5

Reference Frame

Figure 2.1



(a) Two-dimensional coordinate system—could be used to represent motion in two dimensions.

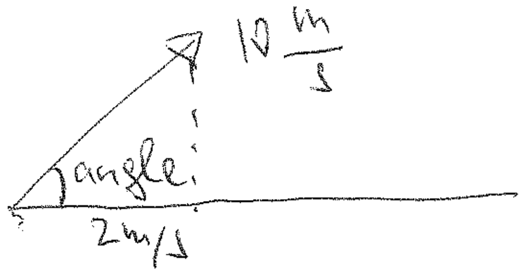


(b) Three-dimensional coordinate system—could be used for motion in three dimensions.

Position

- x is the position coordinate on a number line called the x-axis.
  - Position is a function of time, i.e., $x = x(t)$
  - Position coordinate may be positive or negative
  - Origin of position coordinate ($x = 0$) is arbitrary. Select location of origin to make problem simple.
- t is time coordinate.
  - Time coordinate may be positive or negative.
  - Origin of time coordinate ($t = 0$) is arbitrary. Select $t = 0$ to make problem simple.

Peter runs up a hill at 10.0 m/s. The horizontal component of Peter's velocity vector was 2 m/s. What was the angle of the hill?



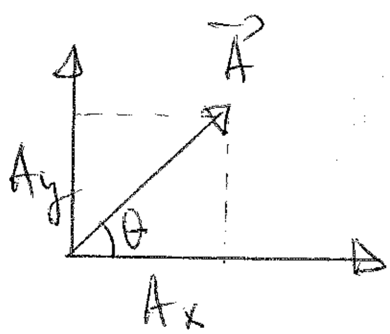
$\text{Cos(angle)} = 2\ [\text{m/s}]/10.0\ [\text{m/s}] = 0.2$

$\text{angle} = \text{Cos}^{-1}(0.2) = 78.5\ \text{deg.}$

Vectors in Terms of Unit Vectors

	Unit vectors	Scalar components
x-direction	$\hat{i}$	a_x
y-direction	$\hat{j}$	a_y
z-direction	$\hat{k}$	a_z

$$\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$



$A_x = \cos\theta\, A$, $tg\,\theta = \dfrac{A_y}{A_x}$

$A_y = \sin\theta\, A$

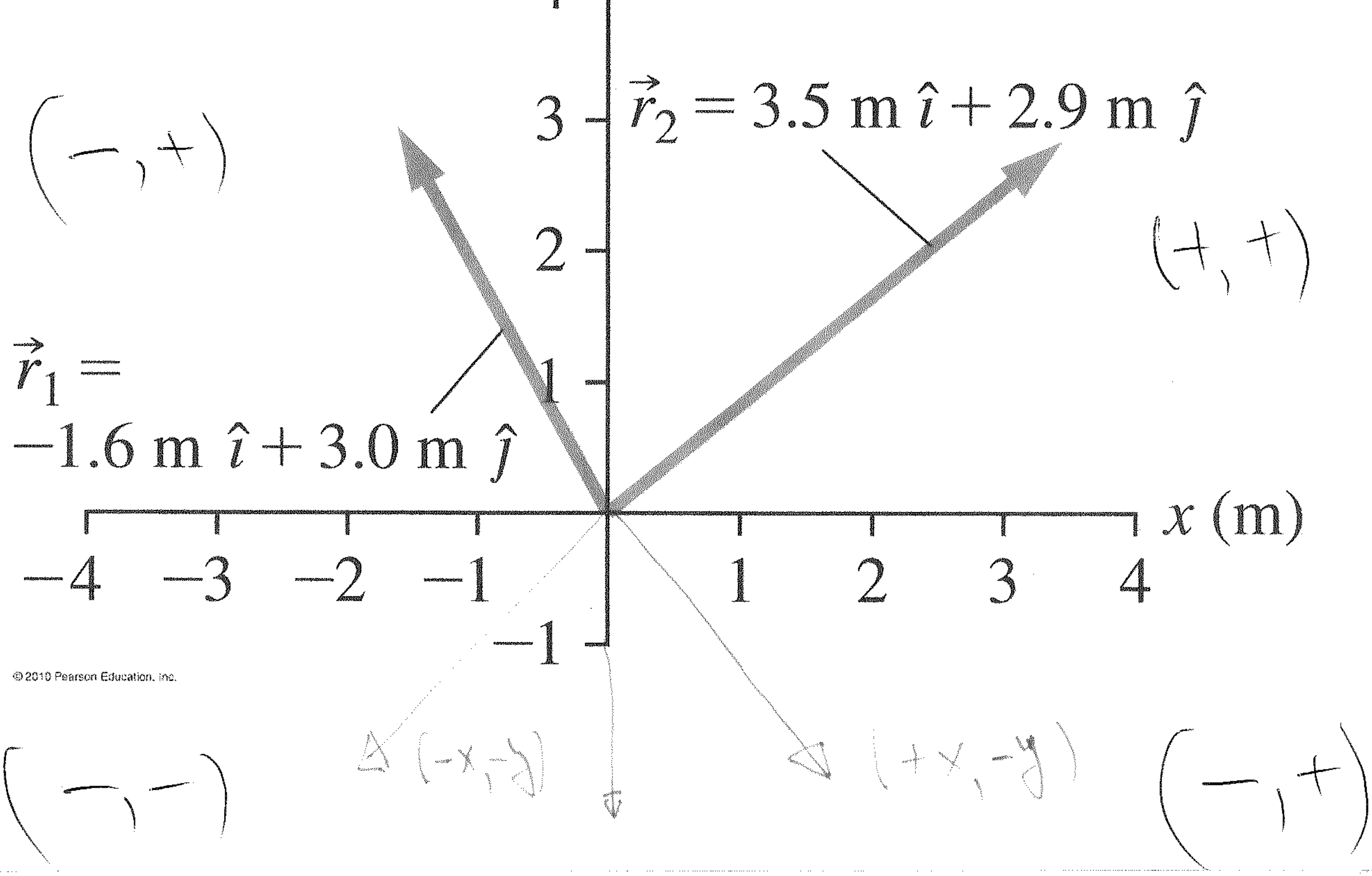
$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

Figure 3.5

Adding and Subtracting Vectors

$\vec{r}_2 + \vec{r}_1 = ?$

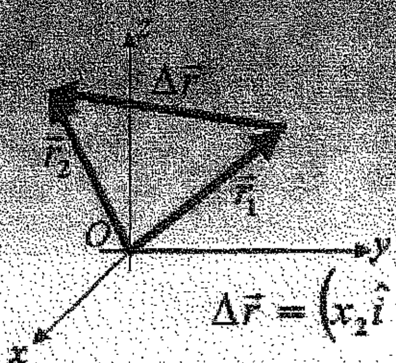
$\vec{r}_2 - \vec{r}_1 = ?$



3

Displacement

The displacement $\Delta\vec{r}$ of a particle is the difference of two position vectors.



$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\Delta\vec{r} = \left(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}\right) - \left(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}\right)$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$$

Displacement

$$\Delta X = X - X_0$$

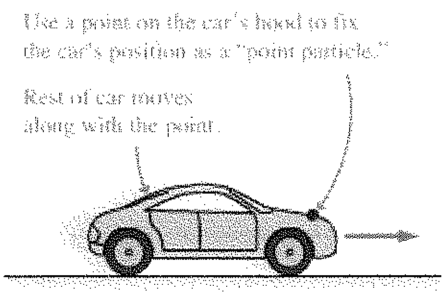


FIGURE 2.3 A real object (a car) modeled as a point particle.

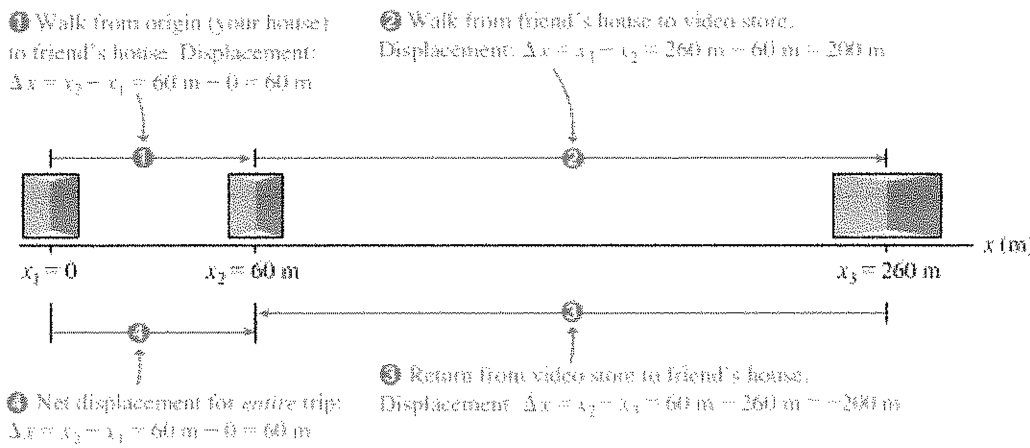


FIGURE 2.4 A trip illustrating displacement in one dimension.

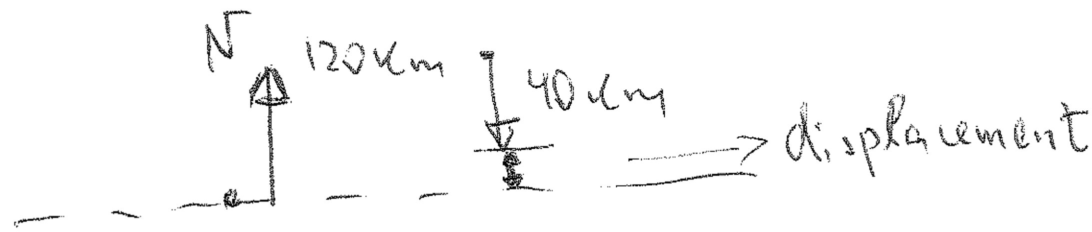
Distance = 200 m + 60 m = 320 m

Displacement = 60 − 0 = 60 m

$$\vec{V} = \overrightarrow{\text{Velocity}} = \frac{\overrightarrow{\Delta X}}{\Delta t}$$

$$\text{Speed} = \frac{\text{Distance}}{\Delta t}$$

A car travels 120 km to the north at 60.0 km/h, then turns around and travels 40 km at 80.0 km/h. What is the difference between the average speed and the average velocity on this trip?



t_1 = 120 [km]/ 60 [km/h] = 2 h +
t_2 = 40 [km]/80 [km/h] = 0.5 h
Total time = 2.5 h

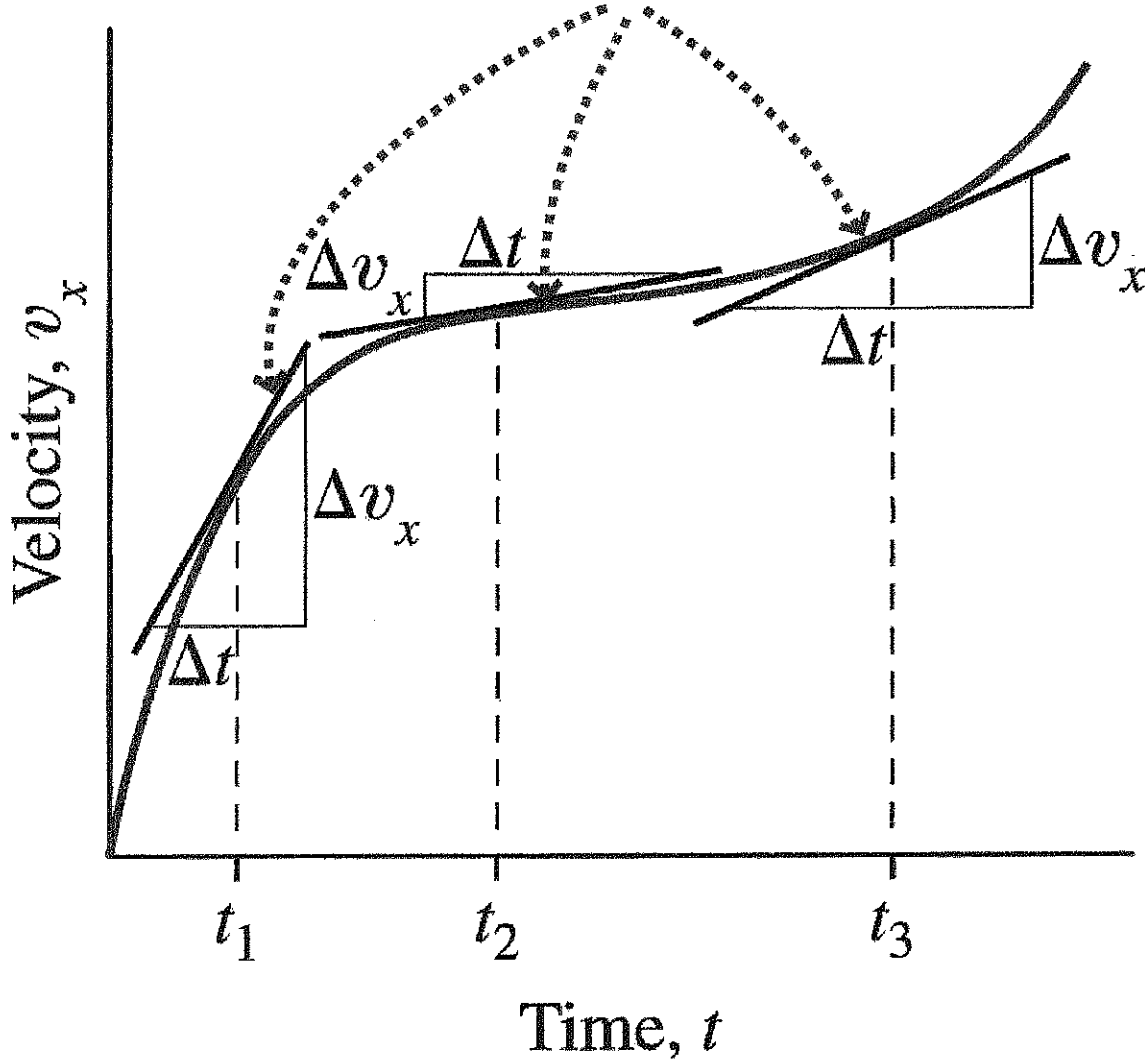
Total journey = 120 km + 40 km = 160 km
Speed = Total journey/total time = 64 km/h

Displacement = 120 km – 40 km = 80 km
Average velocity = 80 [km]/2.5 h = 32 km/h

Difference (speed- velocity) = 32 km/h

Figure 2.14

The slopes of three tangent lines give the instantaneous acceleration at three different times.



$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{m}{s^2}$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$$

$$a \to \left[\frac{m}{s^2}\right]$$

A rocket accelerates from 1 m/s to 100 m/s at a rate of 10 m/s^2. How far does it travel when accelerating ?

$$V = V_o + a.t$$

$$t = (V-V_o)/a = 9.9 \text{ sec}$$

$$X = X_o + V_o.t + ½\, a.t^2$$

$$X - X_o = V_o.t + ½\, .a.t^2 = 499.95 \text{ m}$$

Projectile Motion

- The particle is launched with an initial velocity $\vec{v}_0$ at an angle θ_0 with respect to the horizontal.

$$\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j}$$

$$v_{0x} = v_0 \cos\theta_0 \qquad v_{0y} = v_0 \sin\theta_0$$

- At any point along the particle's trajectory, the particle has a velocity $\vec{v} = v_x\hat{i} + v_y\hat{j}$

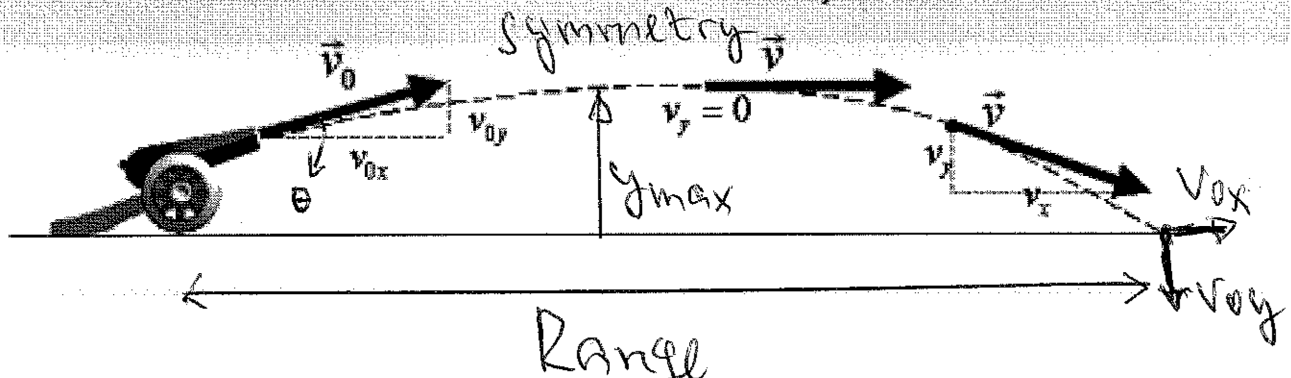


Table 3.2

TABLE 3.2 Kinematic Equations in Two Dimensions

Kinematic equations for x		For projectile, with $a_x = 0$	
$x = v_{0x}t + \frac{1}{2}a_x t^2$	(2.9)	$x = v_{0x}t$	(3.18a)
$v_x = v_{0x} + a_x t$	(2.8)	$v_x = v_{0x}$	(3.18b)
$v_x^2 = v_{0x}^2 + 2a_x\Delta x$	(2.10)	$v_x = v_{0x}$	

Kinematic equations for y		For projectile, with $a_y = -g$	
$y = v_{0y}t + \frac{1}{2}a_y t^2$		$y = v_{0y}t - \frac{1}{2}gt^2$	(3.19a)
$v_y = v_{0y} + a_y t$		$v_y = v_{0y} - gt$	(3.19b)
$v_y^2 = v_{0y}^2 + 2a_y\Delta y$		$v_y^2 = v_{0y}^2 - 2g\Delta y$	(3.19c)

Projectile Range

The range R of a projectile is the horizontal distance it travels when its final height equals its initial height.

$$x - x_0 = R = v_0 t \cos\theta = \frac{V_0\cos\theta\, V_0\sin\theta \cdot 2}{g} = \frac{2V_xV_y}{g}$$

$$y - y_0 = 0 = -\tfrac{1}{2}gt^2 + v_0 t \sin\theta$$

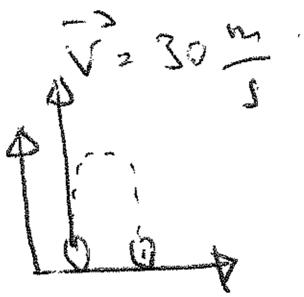
Eliminate *t* from these equations.

$$\frac{1}{2}g\cancel{t}^2 = V_0 \cancel{t} \sin\theta$$

$$t = \frac{V_0\sin\theta}{} \cdot \frac{2}{g}$$

$$R = V_0\cos\theta \cdot t = \frac{2V_0\cos\theta \cdot V_0\sin\theta}{g} = \frac{2V_{0x}\cdot V_{0y}}{g}$$

A ball is thrown straight upward with a velocity of 30 m/s. How much time passes before the ball strikes the ground?



$$Y = Y_0 + V_{0y}*t - (1/2)*g*t^2$$

$$0 = Y - Y_0 = V_{0y}*t - (1/2)*g*t^2$$

$$V_{0y}*t = (1/2)*g*t^2$$

$$V_{0y} = (1/2)*g*t$$

$$\boxed{(V_{0y}*2)/g = t}$$

$$(30*2)/9.82 = t$$

$$6.1 \text{ sec} = t$$

Time for $y = y_{max}$

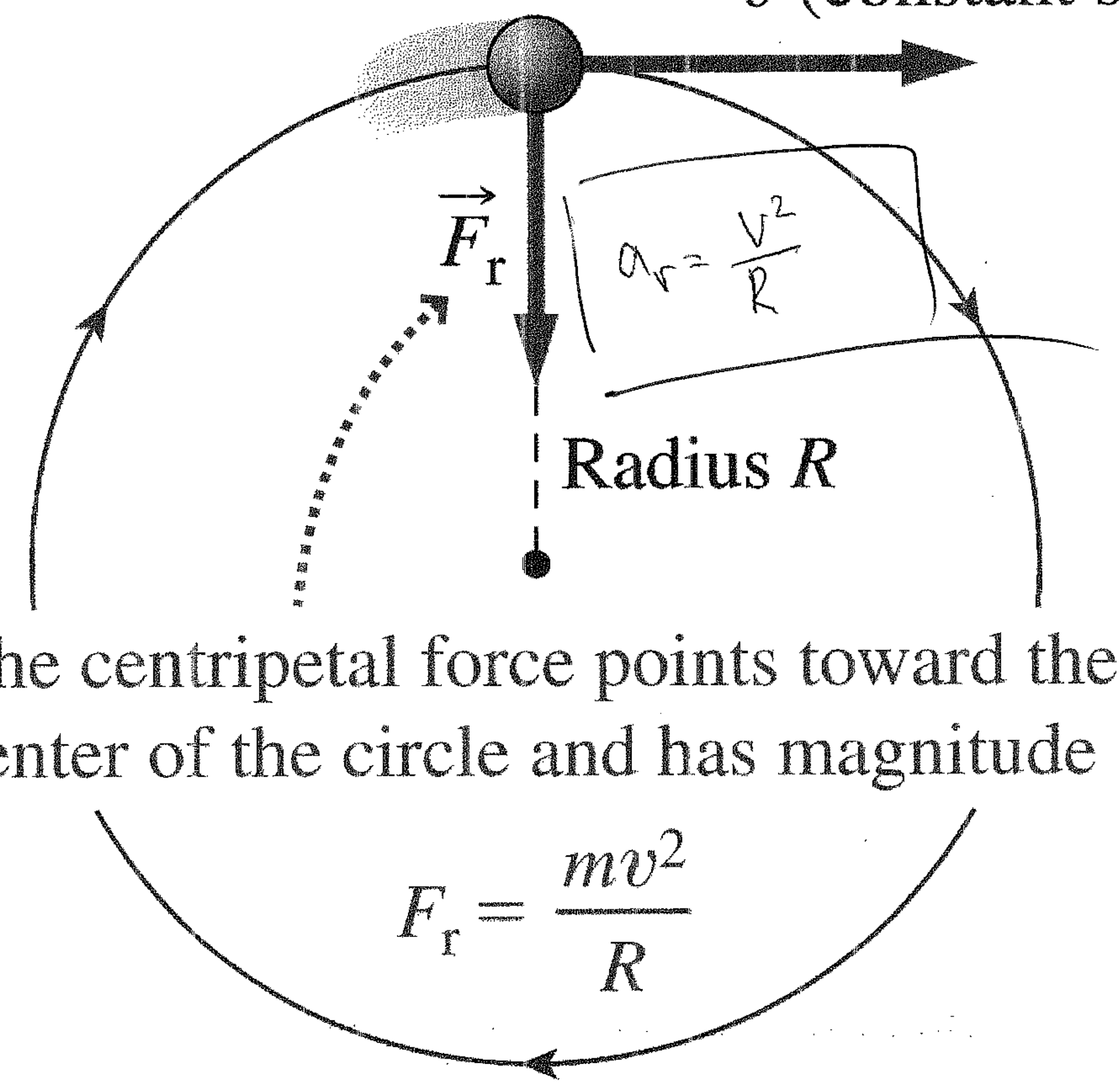
$$t_{y=y_{max}} = \frac{t_{\text{strike Ball}}}{2} = \frac{V_{0y}}{g}$$

Kinematic equations for constant acceleration:

$v_x = v_{x0} + a_x t$	(Predicts velocity; SI unit: m/s)
$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$	(Predicts position; SI unit: m)
$v_x^2 = v_{x0}^2 + 2 a_x \Delta x$	(Relates final and initial velocities, acceleration, and displacement)

Circular motion

Figure 4.27



The centripetal force points toward the center of the circle and has magnitude

$$F_r = \frac{mv^2}{R}$$

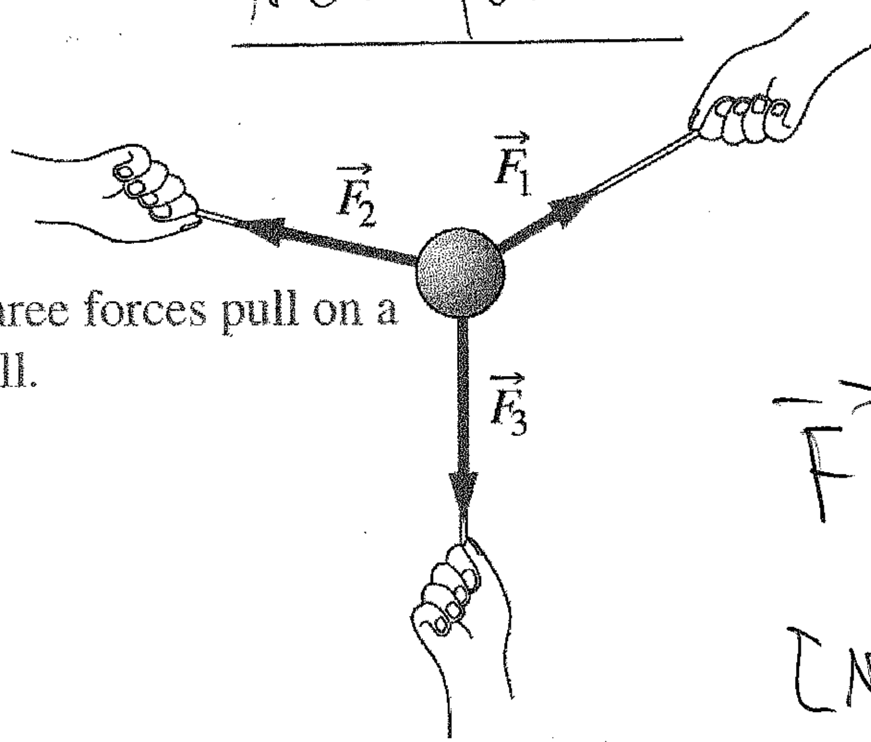
(8)

Applying Newton's Laws

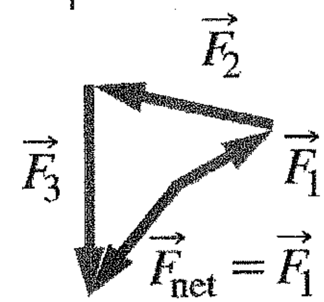
- Identify the system.
- Identify the external forces.
- Draw the free body diagram(s).
- Write Newton's 2nd Law equations. Apply Newton's other laws as necessary.
- Solve the equations for the forces and accelerations.
- ***Check the solution*** by checking special cases.

Figure 4.2

Net force



Three forces pull on a ball.



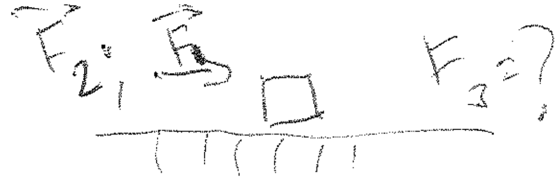
The vector sum of the three forces is the *net force* acting on the ball.

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

The ball behaves as though only the net force acts on it.

$$\vec{F} = m \cdot \vec{a}$$

$$[N] = kg \cdot \frac{m}{s^2}$$



30. **Organize and Plan** Recall that the net force is the vector sum of all forces acting on an object. Two non-zero forces are acting on the box, and we must add a third force to cancel these forces and render the net force zero. If we call the third force $\vec{F}_3$, we just insert it into the expression for the net force and solve for it.

Known: $\vec{F}_1 = -407\ \text{N}(\hat{i}) - 650\ \text{N}(\hat{j})$; $\vec{F}_2 = 257\ \text{N}(\hat{i}) - 419\ \text{N}(\hat{j})$

Solve The net force, which we want to be zero, is given by [Eq.1]: $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$. Inserting the know values for $\vec{F}_1$ and $\vec{F}_2$, we can solve for $\vec{F}_3$ [Eq. 2]:

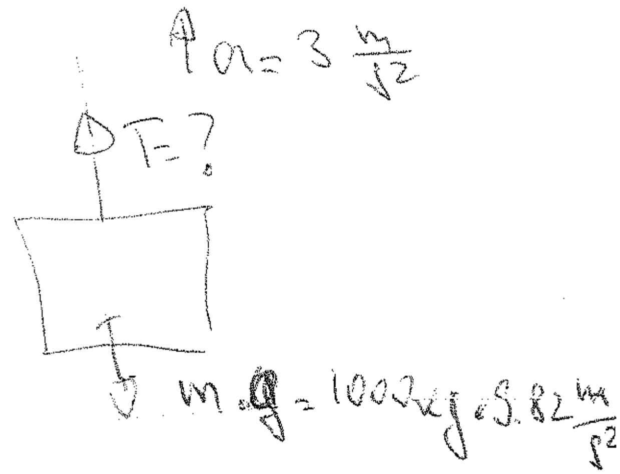
$$\vec{F}_{\text{net}} = 0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$
$$\vec{F}_3 = -\vec{F}_1 - \vec{F}_2$$
$$\vec{F}_3 = (-407\ \text{N} + 257\ \text{N})(-\hat{i}) + (-650\ \text{N} - 419\ \text{N})(-\hat{j})$$
$$\vec{F}_3 = 150\ \text{N}(\hat{i}) + 1069\ \text{N}(\hat{j})$$

We therefore need to apply a force $\vec{F}_3 = +150\ \text{N}(\hat{i}) + 1069\ \text{N}(\hat{j})$ to reduce the net force to zero.

Reflect Note that to construct an anti-parallel vector of equal magnitude to a known vector, it suffices to multiply each component of the known vector by -1.

36. A 1000-kg elevator is rising and its speed is increasing at 3 m/s^2. The tension in the elevator cable is:

1) 1000 N
2) 3000 N
3) 9800 N
4) 12800 N



Ans 4:

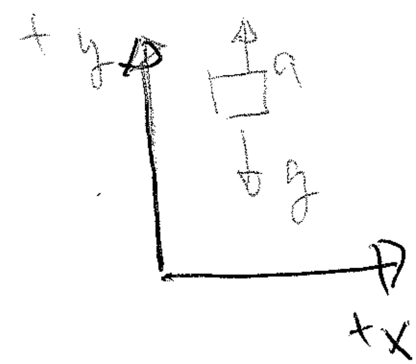
$$\overbrace{ma}^{F_{NET}} = T - mg$$

$$T = ma + mg$$

$$= m(a+g)$$

$$= 1000kg(3 + 9.82)$$

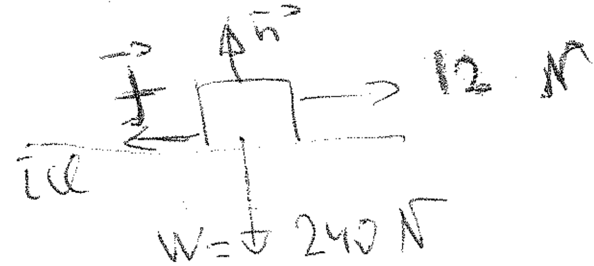
$$= 12{,}800\ N$$



2

4. A forward force of 12 N is used to pull a 240-N sled at constant velocity on a frozen pond. The coefficient of friction is:

1) 0.5
2) 0.05
3) 2
4) 0.2



along $\vec{x}$ ⟶ $12N - f_k = m a = 0$

$12N = f_k = \mu_k n$; $|\vec{n}| = |\vec{w}| = 240N$

$= \mu(240N)$ along $\vec{y}$ direction

$\frac{12N}{240N} = \mu$

$0.05 = \mu$

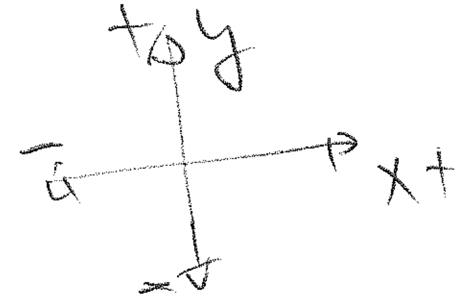
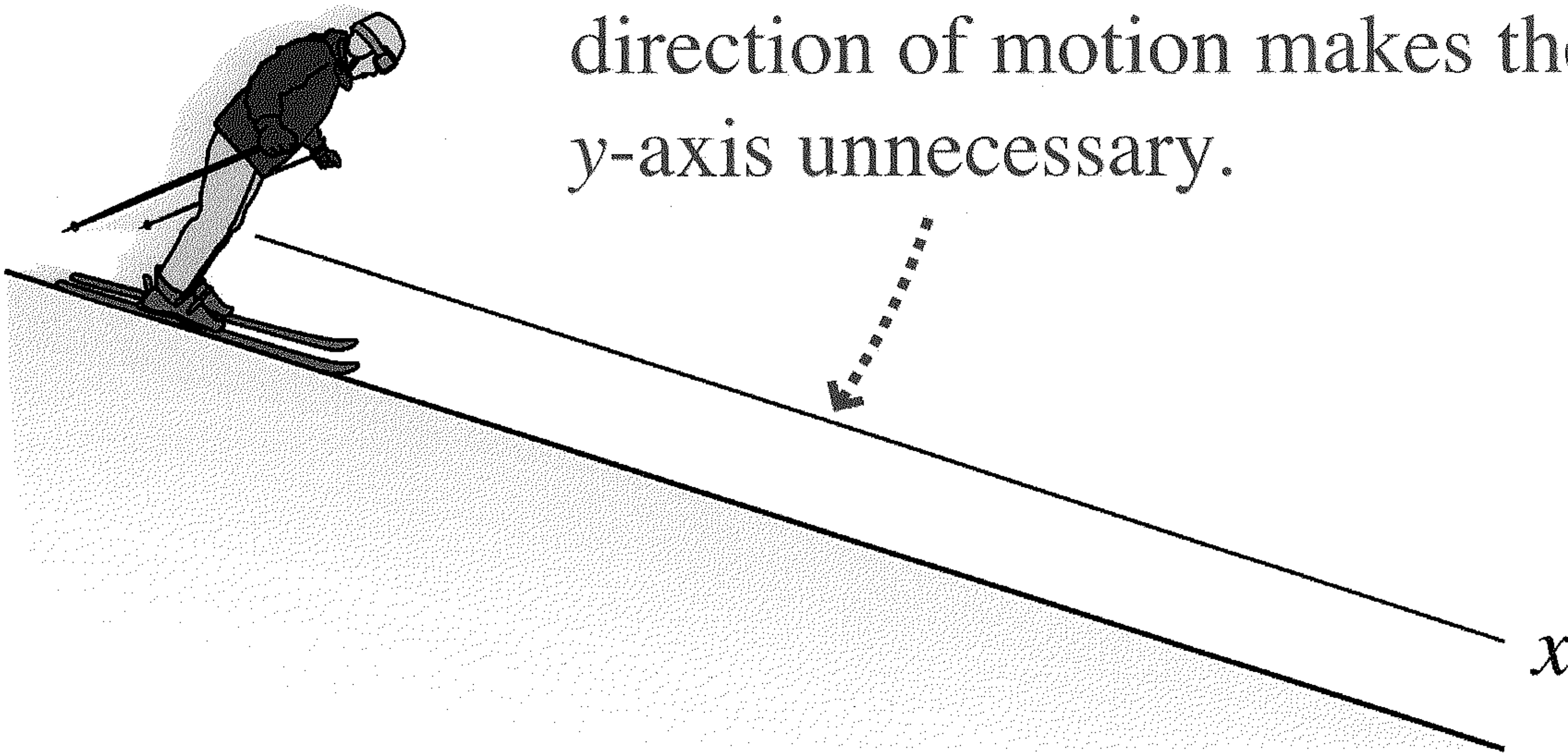


Figure 2.2C

Wise choice of reference frame

Choosing an x-axis in the direction of motion makes the y-axis unnecessary.

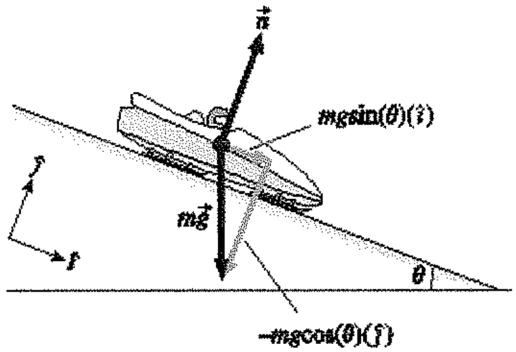


(c) Coordinate system for skier on slope

No friction

$\theta = ?$ So $V = 40 \frac{m}{s}$ for 30 sec

59. **ORGANIZE AND PLAN** See the figure below for a force diagram of the situation. Sum the forces on the sled to find the net force, and then use Newton's second law to find the acceleration due to that force. Chose the incline angle appropriately so that this acceleration gives the desired velocity.



Known: $\vec{v}_0 = 0$ ms; $\vec{v}_f = 40$ m/s$(\hat{i})$; $t = 30$ s

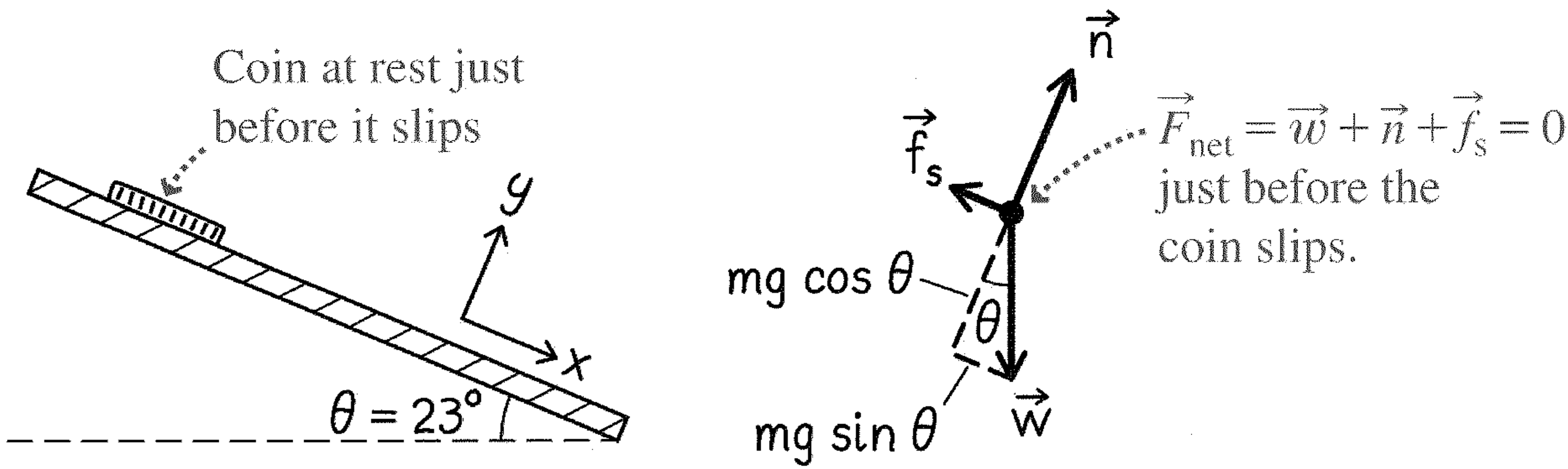
SOLVE Since the sled presumably does not accelerate off the surface of the ice, the forces in the $\hat{j}$ direction must sum to zero (Newton's second law). Therefore the net force on the sled is [Eq. 1] $\vec{F}_{net} = mg\sin(\theta)(\hat{i}) = m\vec{a}$. So [Eq. 2] $\vec{a} = g\sin(\theta)(\hat{i})$.

Inserting this into the equation of motion gives [Eq. 3]

$$\vec{v}_f = \vec{v}_0 + \vec{a}t$$
$$g\sin(\theta)(\hat{i}) = \vec{v}_f / t \quad \rightarrow \sin\theta = \frac{V}{gt}$$

$$\theta = \sin^{-1}\left(\frac{V}{gt}\right) = \sin^{-1}\left(\frac{40\text{ m/s}}{(9.8\text{ m/s}^2)(30\text{ s})}\right) = 7.8°$$

Figure 4.24



$mg\sin\theta - \mu_s n = 0$

$n - mg\cos\theta = 0 \rightarrow n = mg\cos\theta$

$\cancel{mg}\sin\theta - \mu_s \cancel{mg}\cos\theta = 0$

$\sin\theta = \mu_s \cos\theta$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \mu_s$