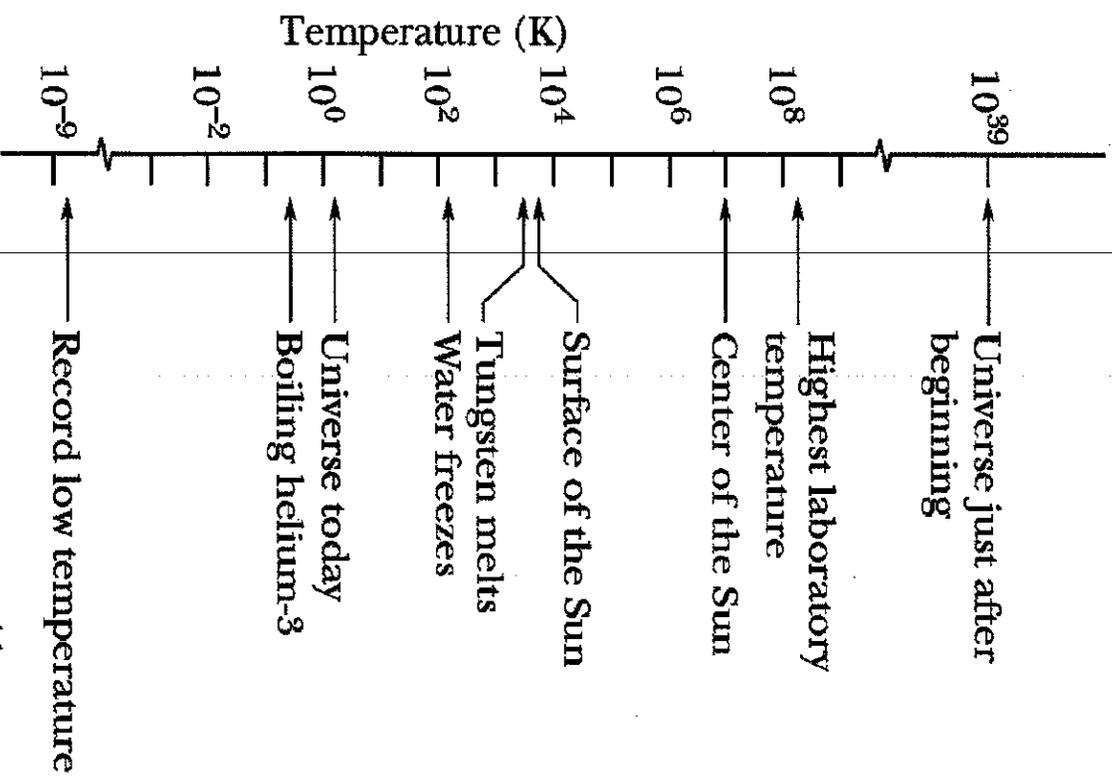


# Thermodynamics & Temperature

- **Mechanics:** mechanical energy of systems governed by Newton's laws
- **Thermodynamics:** internal energy of bodies – *thermal energy*
- **Temperature:** central concept - sometimes a measure of the internal energy
  - Kelvin temperature - absolute temperature (see Fig. 18-1)

T [K]



# Celsius and Fahrenheit Scales

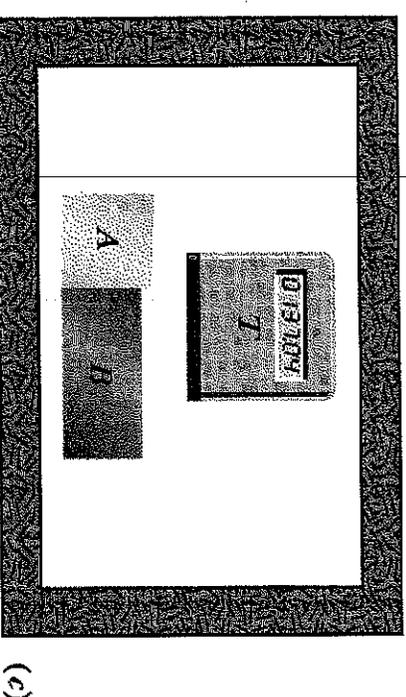
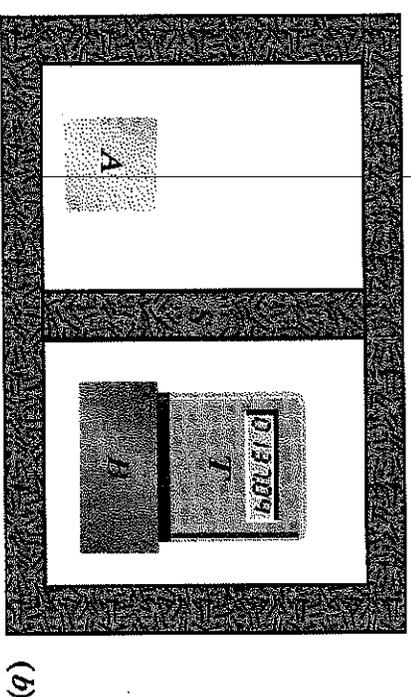
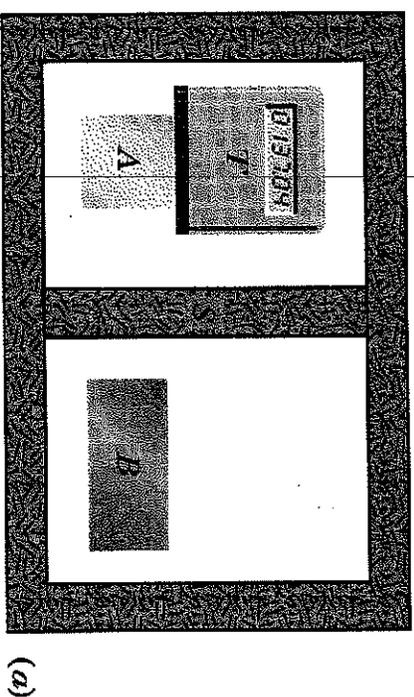
- Celsius scale: used worldwide

$$T_C = T - 273.15 K$$

- $1^\circ \text{C} = 1 \text{K}$
- $0^\circ \text{C} \Rightarrow$  ice-water phase transition, normal pressure
- $100^\circ \text{C} \Rightarrow$  water-vapor phase transition
- $37^\circ \text{C} \Rightarrow$  normal human body temperature
- $20^\circ \text{C} \Rightarrow$  normal room temperature ( $68^\circ \text{F}$ )

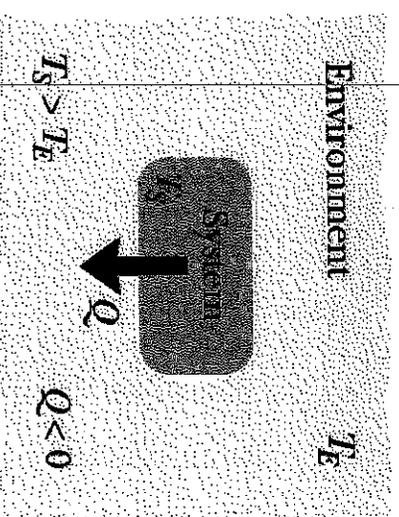
# The Zeroth Law of Thermodynamics

- **Thermal equilibrium:** no net exchange of thermal energy
- **Thermoscop** (see Fig. 18-2)
- **Zeroth Law** (see Fig 18-3 =>):
  - *If bodies A and B are each in thermal equilibrium with a third body T, then A and B are in equilibrium with each other*
  - *Each body characterized by its temperature*

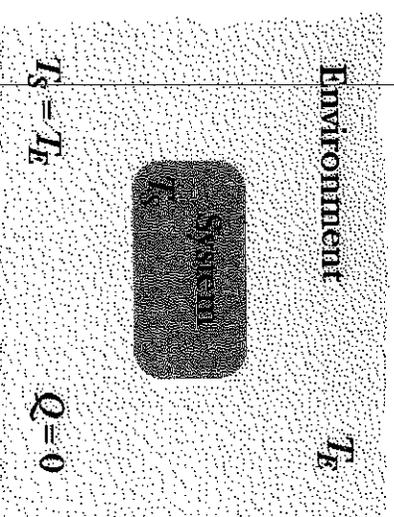


# Temperature and Heat ( $Q$ )

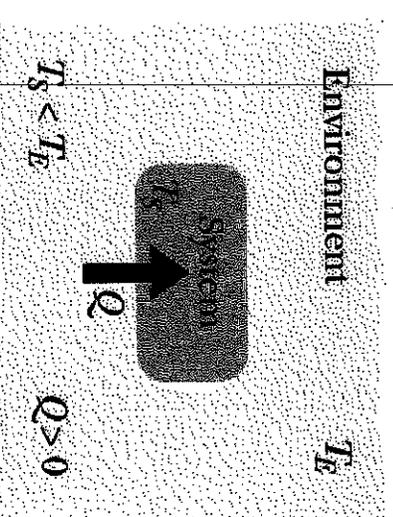
- System vs Environment
- Change in temperature: transfer of internal energy, called **heat**
- (see Fig 18-12 =>)
- **Heat** is the energy that is transferred between a system and its environment because of the temperature difference that exists between them



(a)



(b)



(c)

- **Units of heat (Q):**

- *calorie (cal):* heat 1 gram of water from 14.5° C to 15.5° C

- *British thermal unit (Btu):* heat 1 lb of water from 63° F to 64° F  $1 \text{ Btu} = 1055,05 \text{ J}$

- *Joule (J):* SI unit ;  $1 \text{ cal} = 4,186 \text{ J}$

$1 \text{ cal} = 4,186 \text{ J}$

$1 \text{ Food Calorie} = 1,000 \text{ cal} = 4186 \text{ J}$

# Review: Absorption of Heat

- Heat Capacity  $Q = C(T_f - T_i)$

- Specific Heat  $Q = cm(T_f - T_i)$

- Molar Specific Heat  $Q = C_n(T_f - T_i)$   
*Handwritten:  $C_v$  and  $C_p$  with arrows pointing to  $C_n$*

- Heats of Transformation:  $Q = Lm$

$L_V, L_F$

# Absorption of Heat

- **Heat Capacity**
  - capacity of a body to absorb heat
  - specific to one body

$$Q = C(T_f - T_i)$$

$$C = \frac{Q}{T_f - T_i} \quad \rightarrow \text{[J]} \quad \rightarrow \text{[K]}$$

- Units: cal/K, Btu/K, J/K

# • Specific Heat

– specific to units of mass

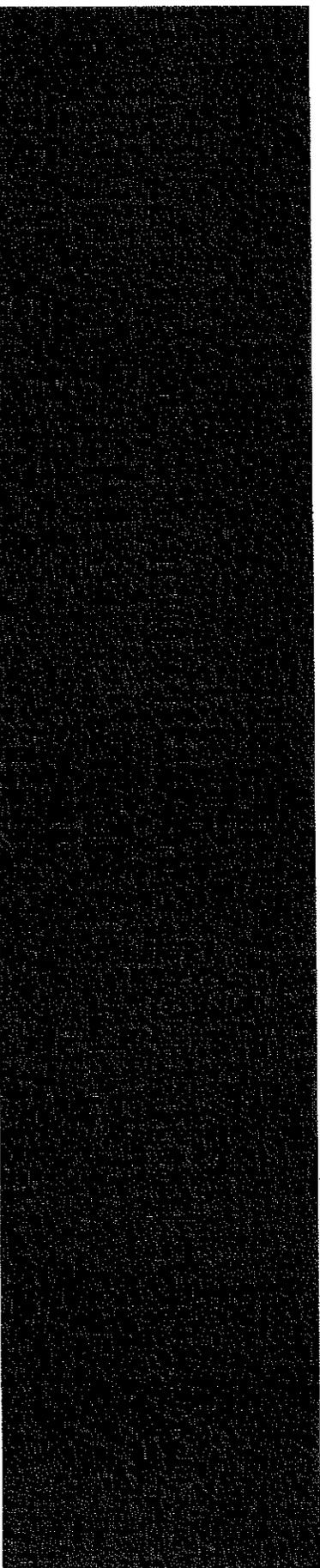
$$Q = cm(T_f - T_i)$$

$Q$  [J],  $m$  [kg],  $c$  – given  
 $\Delta T \rightarrow$  in [K] or [°C]  
does not matter

$$c = \frac{Q}{m\Delta T} = \frac{Q}{m(T_f - T_i)}$$

– specific heat of water (for other Table 18-3)

$$c_w = 1 \text{ cal/g} \cdot \text{K} = 1 \text{ Btu/lb} \cdot \text{F} = 4190 \text{ J/Kg} \cdot \text{K}$$



# • Heats of transformation

Phase changes

- phases or states: solid, liquid, gas
- When a phase transform into another (phase transformation) the temperature is constant
- Needs *heat of transformation*

$$Q = Lm \quad L = \frac{Q}{m}$$

- Units for L: J/Kg
- Examples (Table 18-4); Heat of vaporization  $L_v$ ; Heat of fusion  $L_f$

TABLE 19-4 Some Heats of Transformation

Substance	Melting		Boiling	
	Melting Point (K)	Heat of Fusion $L_f$ (kJ/kg)	Boiling Point (K)	Heat of Vaporization $L_v$ (kJ/kg)
Hydrogen	14.0	58.0	20.3	455
Oxygen	54.8	13.9	90.2	213
Mercury	234	11.4	630	296
Water	273	333	373	2256
Lead	601	23.2	2017	858
Silver	1235	105	2323	2336
Copper	1356	207	2868	4730

# Heat Transfer Mechanisms

- **Conduction:** through solids (metals) due to atomic vibration
  - Conduction through a slab (Fig. 19-18)

$$P_{cond} = \frac{Q}{t} = k A \frac{T_H - T_C}{L}$$

- $P_{cond}$  – conduction rate (per unit time)  $\rightarrow \text{J/s} \rightarrow \text{W}$
- $k$  - thermal conductivity
- $T_H$  ( $T_C$ ) – hot(cold) reservoir temperature

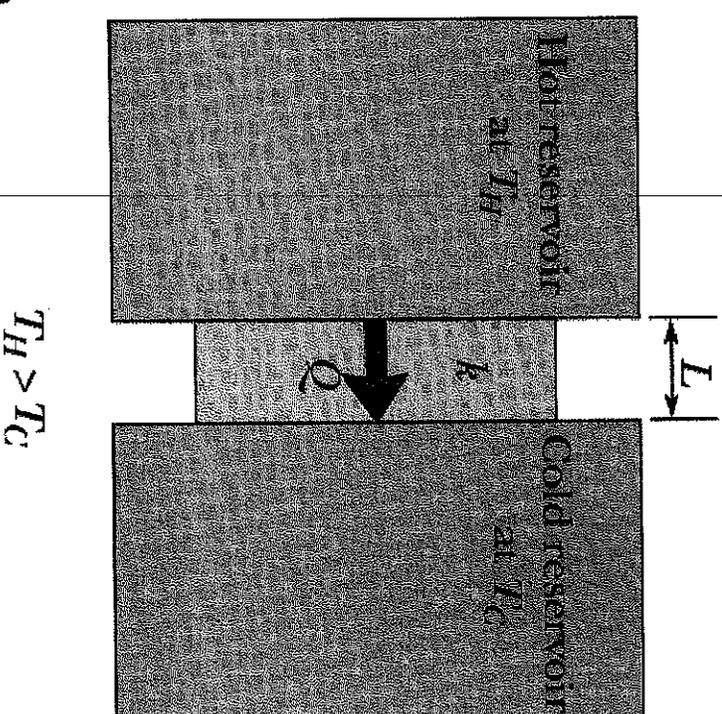
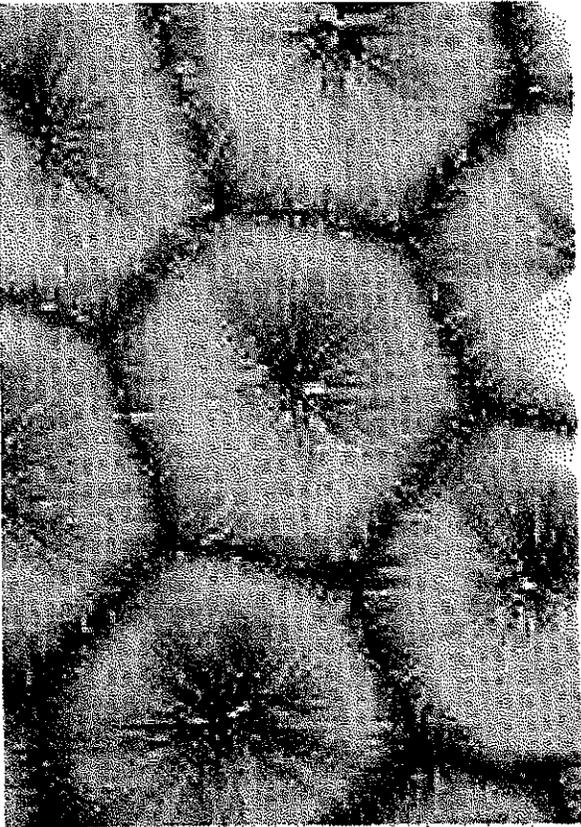
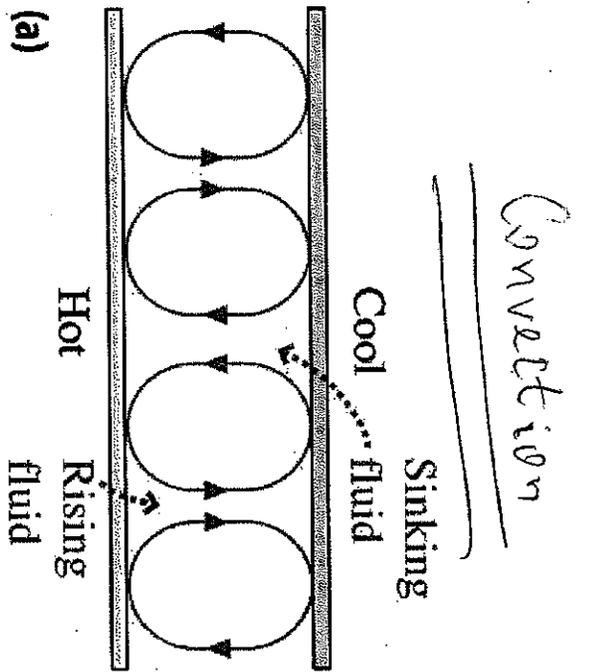
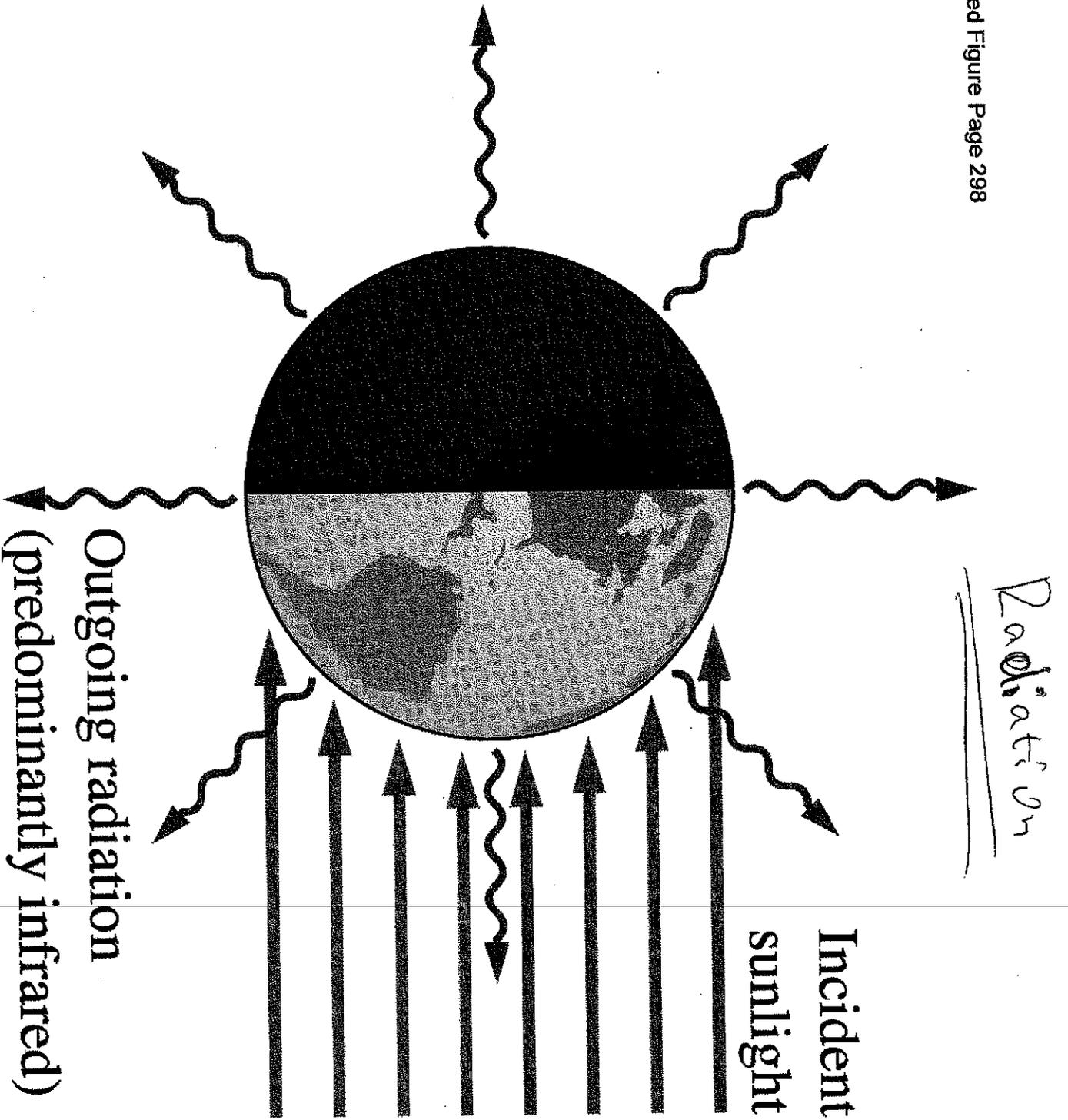


Figure 13.19



(b)



- Convection – atmospheric convection
- Radiation –  $T [K]$

$$P_{\text{rad}} = \sigma \epsilon A T^4 \quad A \rightarrow \text{area } [m^2]$$

where  $\sigma = 5.673 \times 10^{-8} \text{ W/m}^2 \text{ K}$   
 (Stefan-Boltzmann)

$$P_{\text{abs}} = \sigma \epsilon A T_{\text{env}}^4 \quad (\text{env})$$

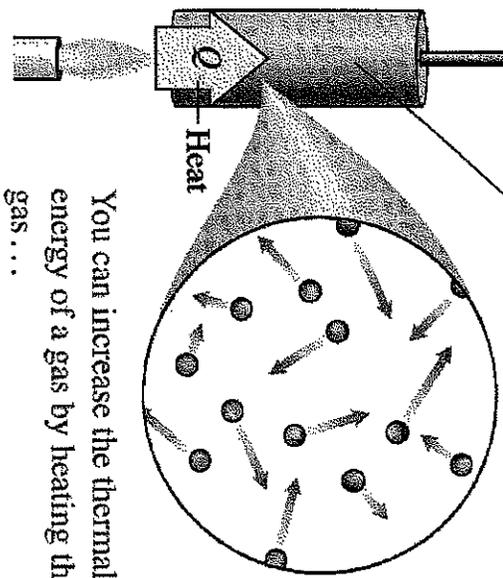
$$P_{\text{net}} = P_{\text{abs}} - P_{\text{rad}} = \sigma \epsilon A (T_{\text{env}}^4 - T^4)$$

$\epsilon$  – emissivity  $[0-1]$

Figure 14.1

# Laws of Thermodynamics

Cylinder of gas



You can increase the thermal energy of a gas by heating the gas...

from  $Q$

1st Law

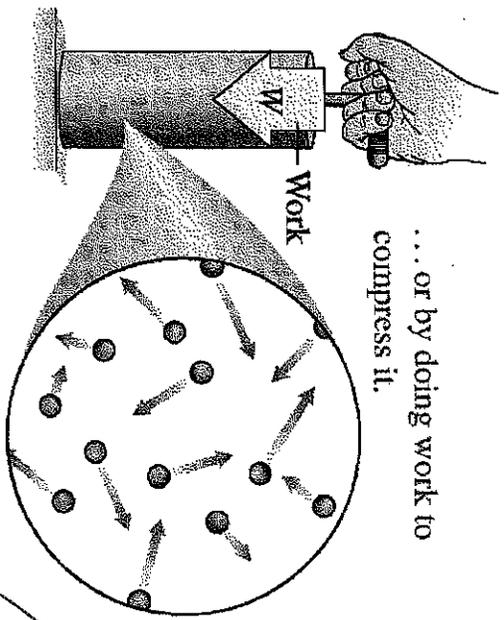
$$\Delta U = Q + W$$

Internal  $\rightarrow E_{\text{kin}}$   
 $\rightarrow E_{\text{pot}}$

no bonds  
 Monatomic gas

$$\Delta U = \Delta E_{\text{kin}} = \frac{3}{2} N k_B \Delta T$$

... or by doing work to compress it.



(b)

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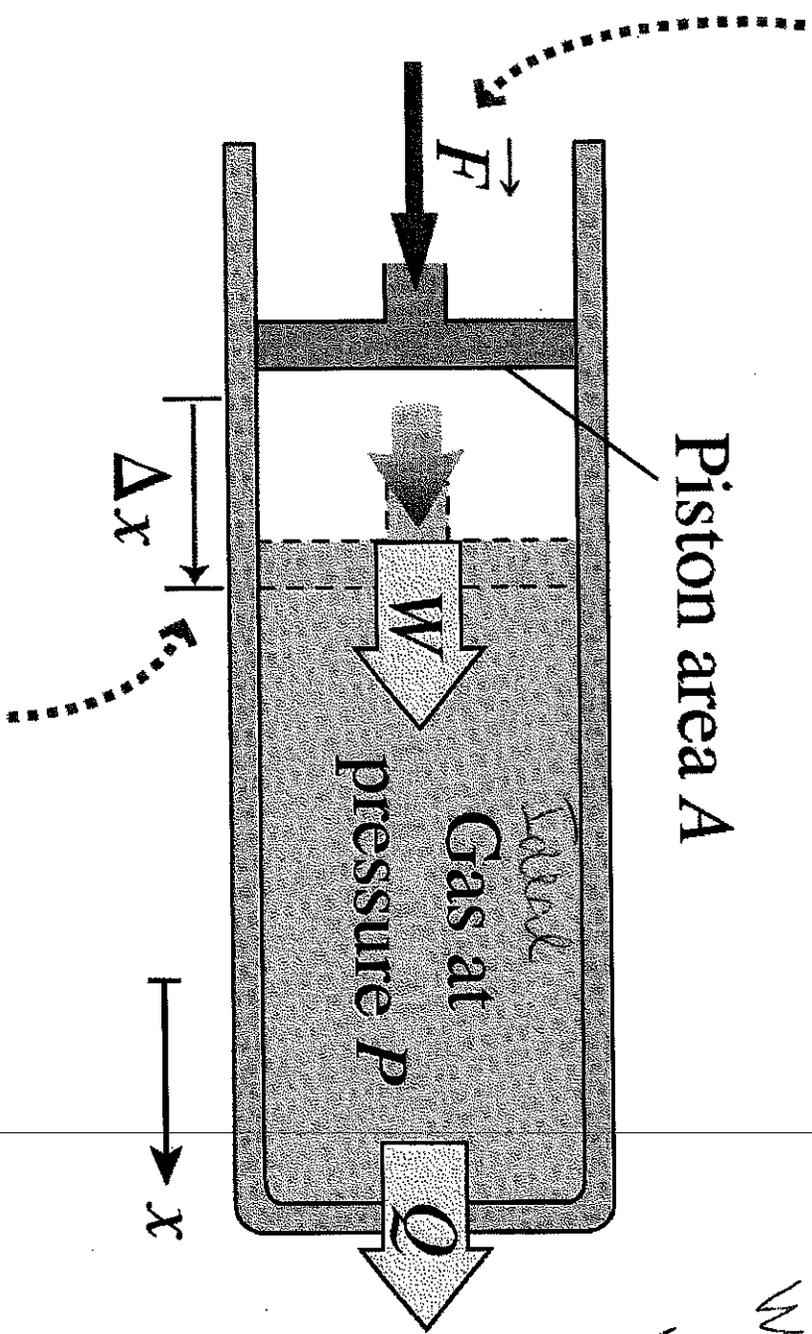
from  $Q$

Diatomic gas  
 single bond  
 + 2 degrees of freedom

$$\Delta U = \Delta E_{\text{kin}} = \frac{5}{2} N k_B \Delta T$$

Figure 14.3

A constant force is applied and heat is allowed to escape, so the pressure remains constant as the gas is compressed.



$P = \text{const.}$  (Isobaric process)

$P = \text{const}$

$$W = F_x \cdot \Delta x = P \cdot A \cdot \Delta x$$

$$V = A \cdot x$$

$$\Delta V = -A \cdot x$$

$$W = -P \cdot \Delta V$$

where

$$\Delta V = V_f - V_i$$

$$\Delta V < 0$$

so  $W > 0$

for fluid

$$\Delta U = Q + W$$

The piston moves through displacement  $\Delta x$ , resulting in work  $W = F_x \Delta x = PA \Delta x$ .

$T = \text{const.}$  Isothermal process

Figure 14.4

In an isothermal process, the temperature of the gas is held constant while the gas is compressed (or expands).

From  $P = nRT/V$   
 $P \cdot V = nR \cdot T$

Fluid bath at constant temperature  $T$

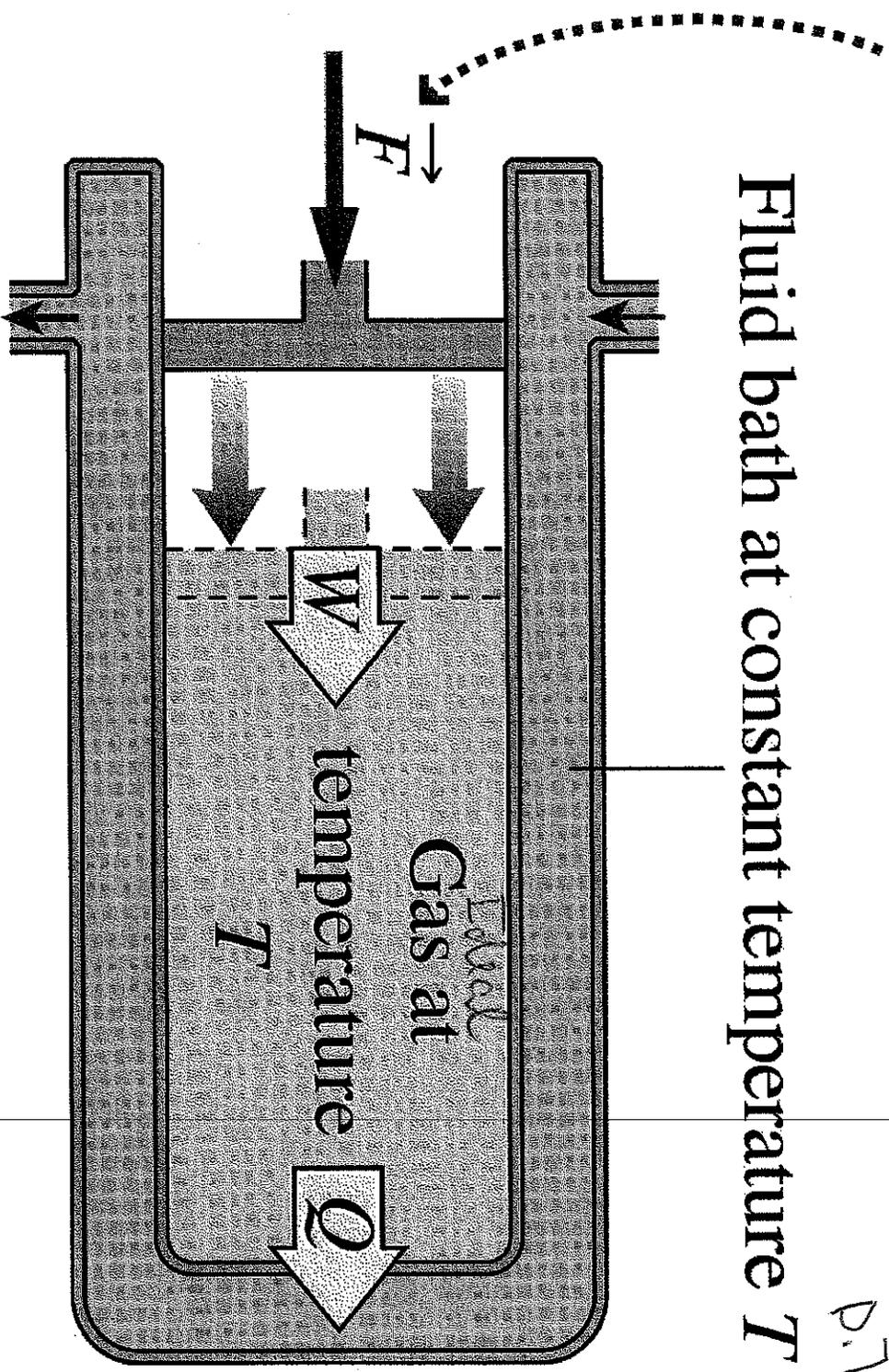
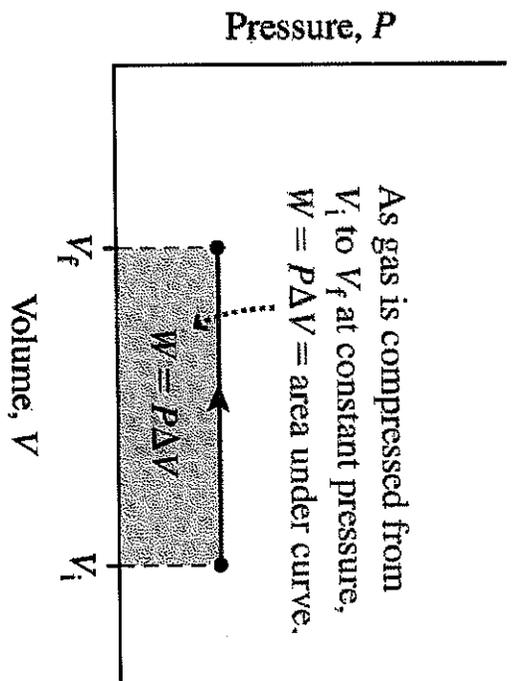
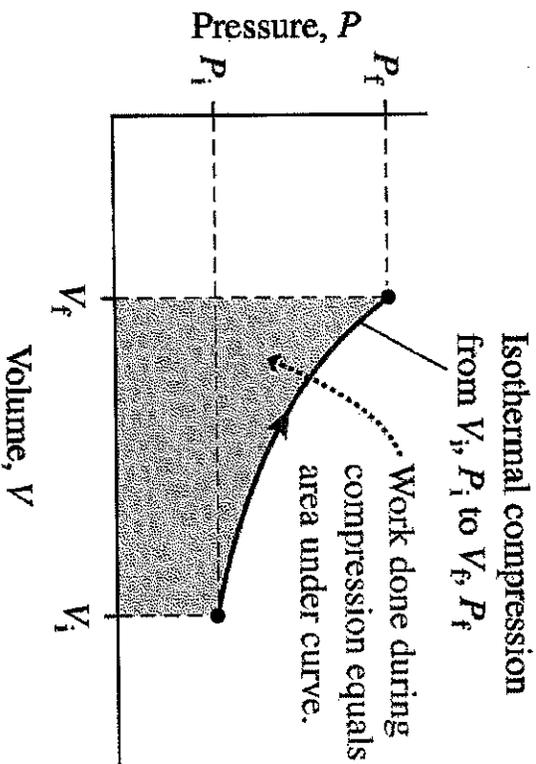


Figure 14.5



(a) Compression at constant pressure



(b) Isothermal compression

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For ideal gas  $T_{\text{const}} \rightarrow \Delta U = 0$  so  $Q + W = 0$

For  $P = \text{const}$

$$W = -P \Delta V$$

$$\Delta V = V_f - V_i < 0$$

so  $W > 0$  for  $\Delta V < 0$

$$P \Delta V = nRT$$

$$P = \frac{nRT}{V}$$

$$W = nRT \ln\left(\frac{V_i}{V_f}\right)$$

for compression

$V_i > V_f$  so  $W > 0$

for expansion

$V_i < V_f$  so  $W < 0$

Figure 14.7

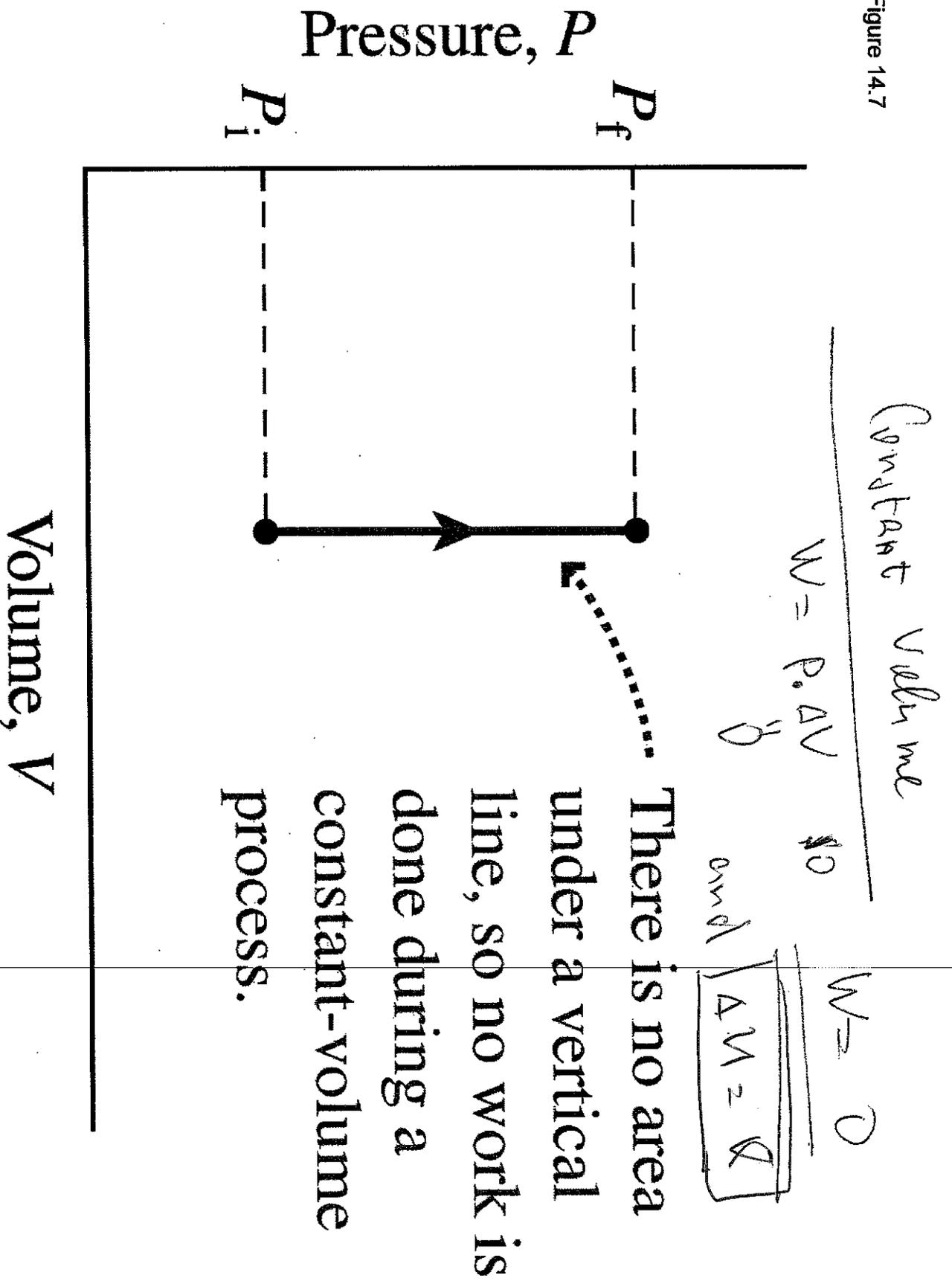


Figure 14.8

Adiabatic Process

$$Q=0$$

Insulation prevents heat flow  
between gas and surroundings.

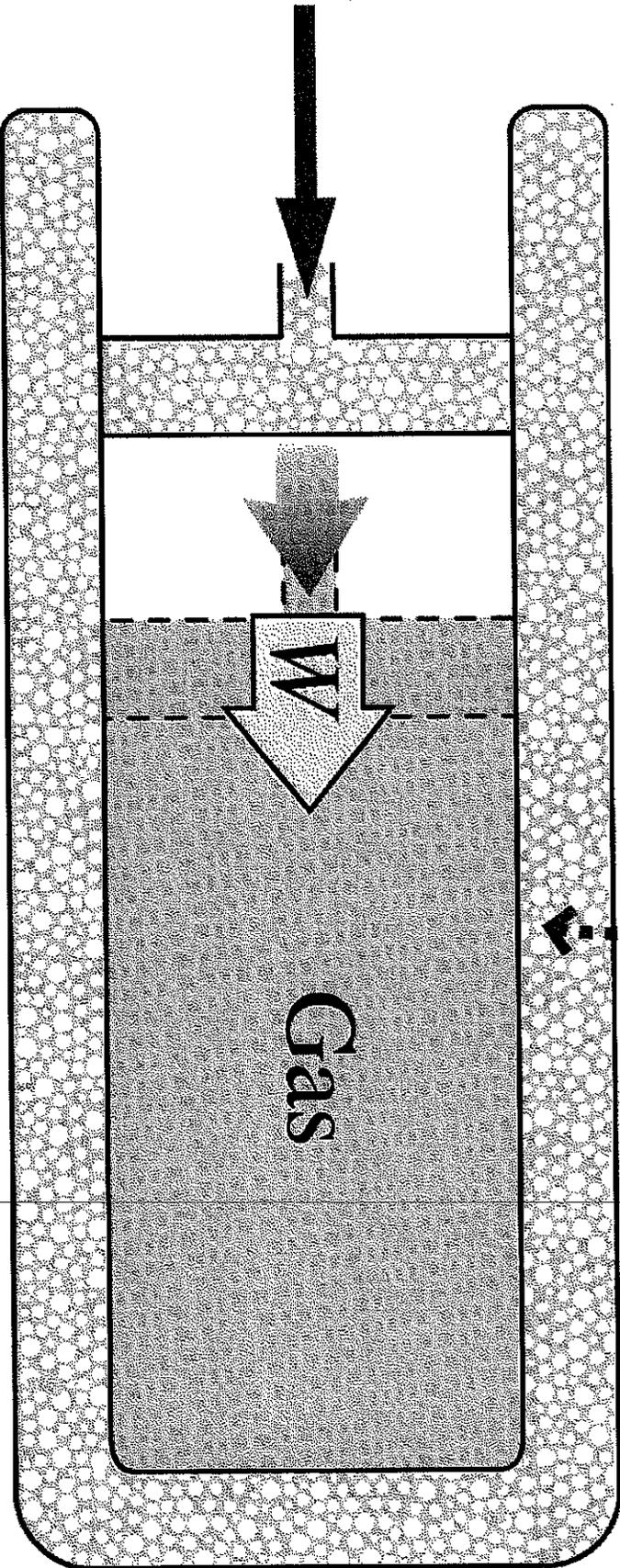
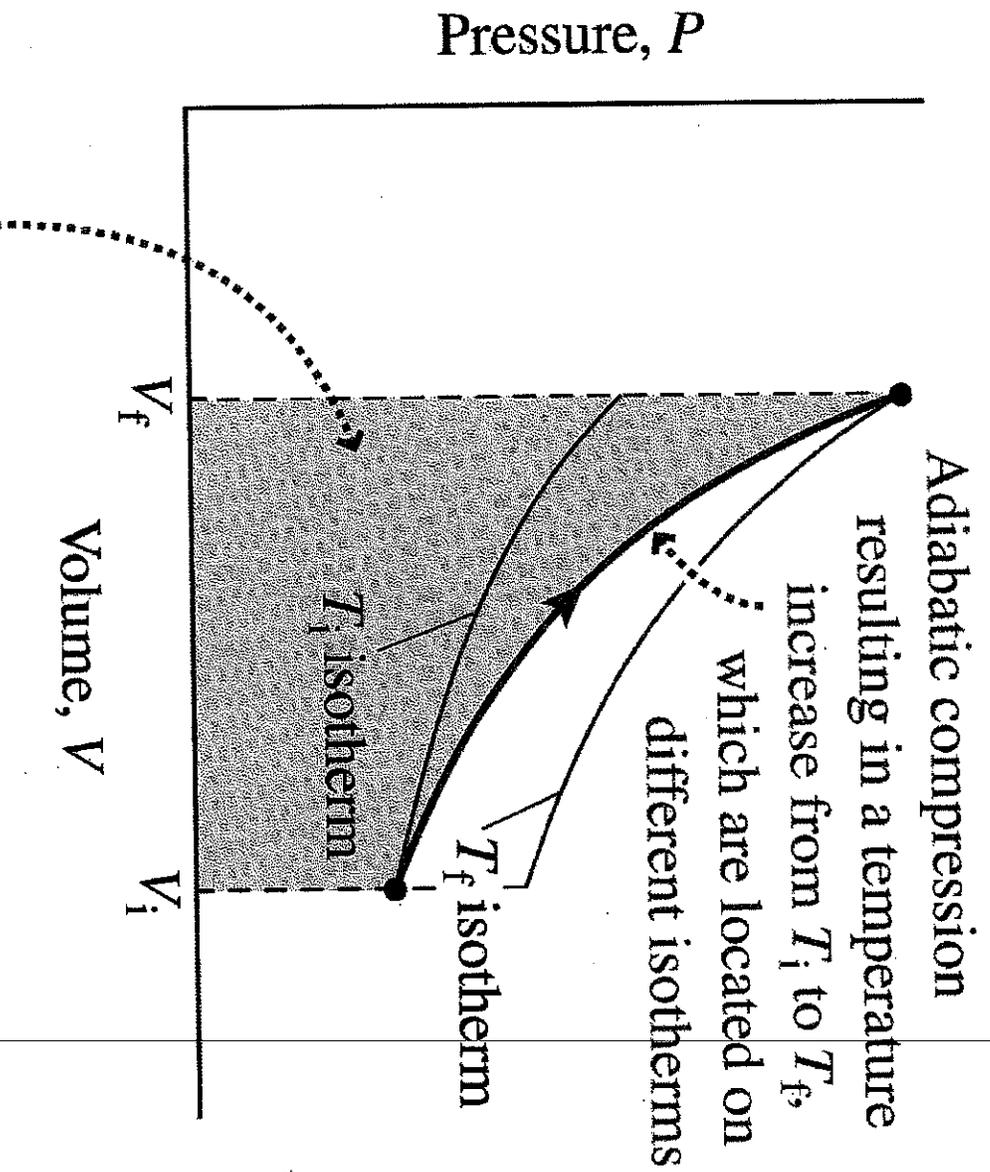


Figure 14.9



Area under adiabatic curve is greater than area under  $T_1$  isotherm, so adiabatic compression requires more work than equivalent isothermal compression.

$$Q = 0$$

$$\text{so } \boxed{\Delta U = W}$$

$$P \cdot V^\gamma = \text{const}$$

$$\gamma = C_p / C_v$$

For monatomic

$$C_p = 5R/2$$

$$C_v = 3R/2$$

$$\text{so } \gamma = 5/3$$

For diatomic

$$\gamma = 7/5$$

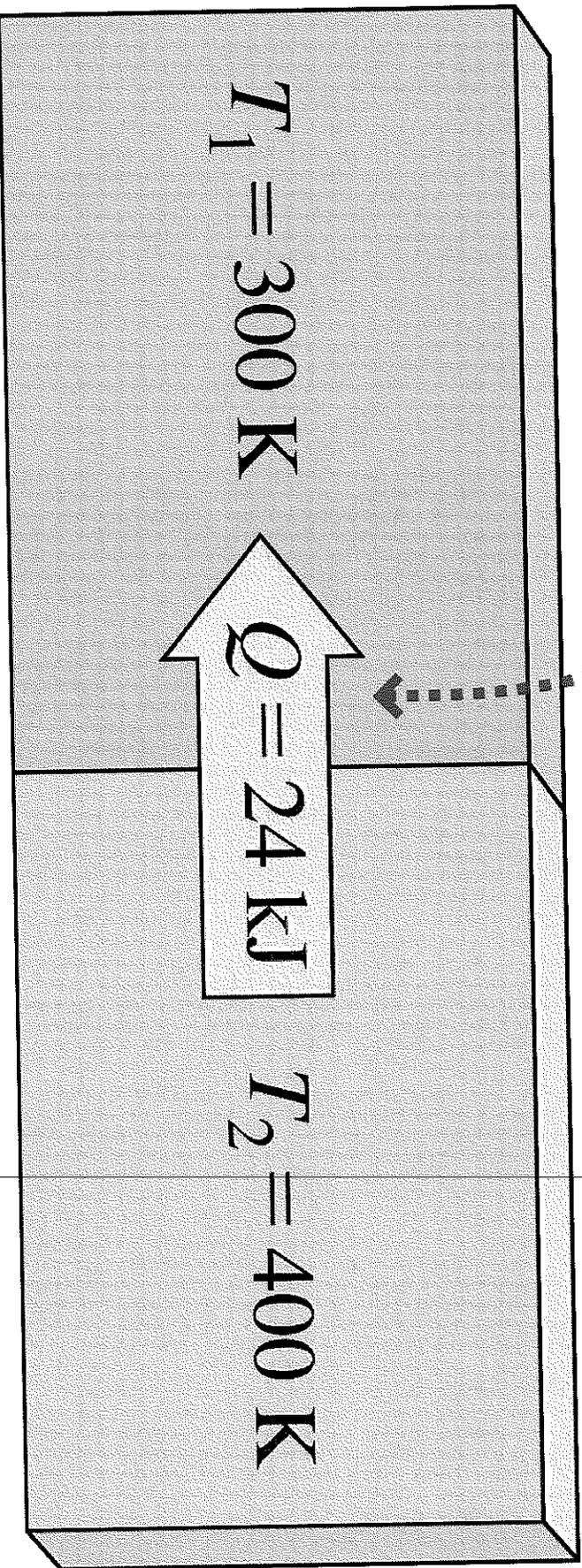
$$W = \frac{P_f V_f - P_i V_i}{\gamma - 1}$$

## Reviewing New Concepts: The First Law of Thermodynamics in Thermal Processes

Process	Work $W$	First law accounting, with $\Delta U = Q + W$
Constant pressure	$W = -P\Delta V$	$\Delta U = Q - P\Delta V$
Constant temperature (isothermal)	$W = nRT \ln\left(\frac{V_i}{V_f}\right)$	$\Delta U = 0$ $Q = -W = -nRT \ln\left(\frac{V_i}{V_f}\right)$
Constant volume	$W = 0$	$\Delta U = Q$
Adiabatic ( $Q = 0$ )	$W = \frac{P_f V_f - P_i V_i}{\gamma - 1}$	$\Delta U = W = \frac{P_f V_f - P_i V_i}{\gamma - 1}$

Figure 14.10

Heat  $Q$  flows from warmer to cooler object.



Second Law (Direction of  
Physical  
Processes, also)

2)

**Second law of thermodynamics:** Heat flows spontaneously only from hotter to cooler objects.

$$\left[ \frac{\text{J}}{\text{K}} \right] \leftarrow \Delta S = \frac{Q}{T} \quad (\text{Entropy change; SI unit: J/K})$$

$\Delta S$  always increases

$\Delta S$  can be  $\pm$  depending on  $\pm Q$

$$\Delta S_{\text{cool}} = \frac{Q}{T_{\text{cool}}} = \frac{24,000 \text{ J}}{300 \text{ K}} = 80 \text{ J/K}$$

Heat flows *out* of the hot object, so  $Q = -24,000 \text{ J}$ , and

$$\Delta S_{\text{hot}} = \frac{Q}{T_{\text{hot}}} = \frac{-24,000 \text{ J}}{400 \text{ K}} = -60 \text{ J/K}$$

The net entropy change of the whole system is

$$\Delta S_{\text{total}} = \Delta S_{\text{cool}} + \Delta S_{\text{hot}} = 80 \text{ J/K} - 60 \text{ J/K} = +20 \text{ J/K}$$

**Second law of thermodynamics:** Heat flow is accompanied by an increase in the entropy of the universe.

**Second law of thermodynamics:** Natural processes evolve toward a state of maximum entropy.

# Heat Engines

- Elements of an engine (Fig 20-7)
  - Heat  $Q_H$  is transferred from the hot reservoir of temperature  $T_H$  to the working substance
  - Heat  $Q_L$  is transferred from the working substance to the cold reservoir  $T_L$

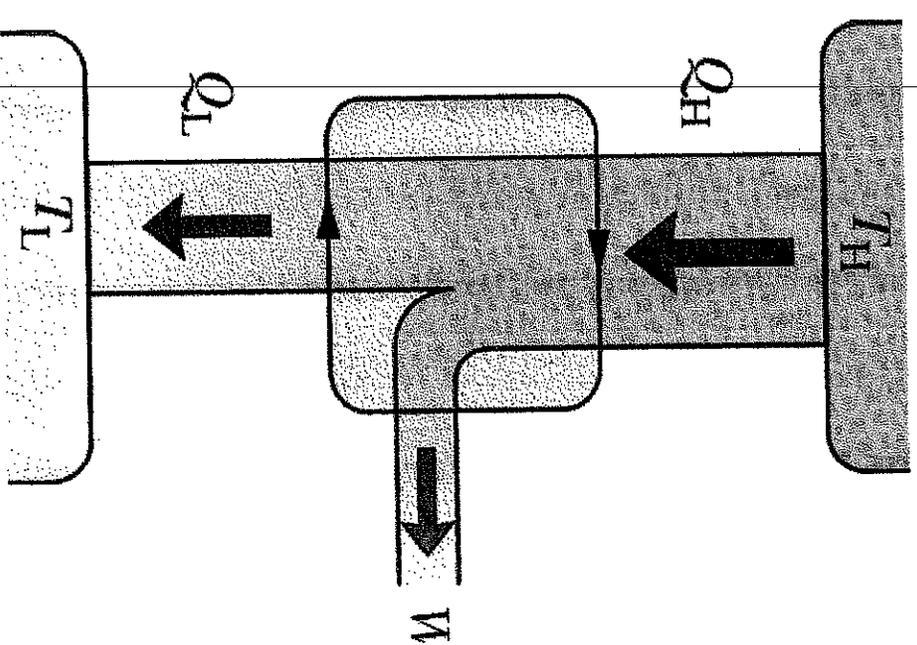
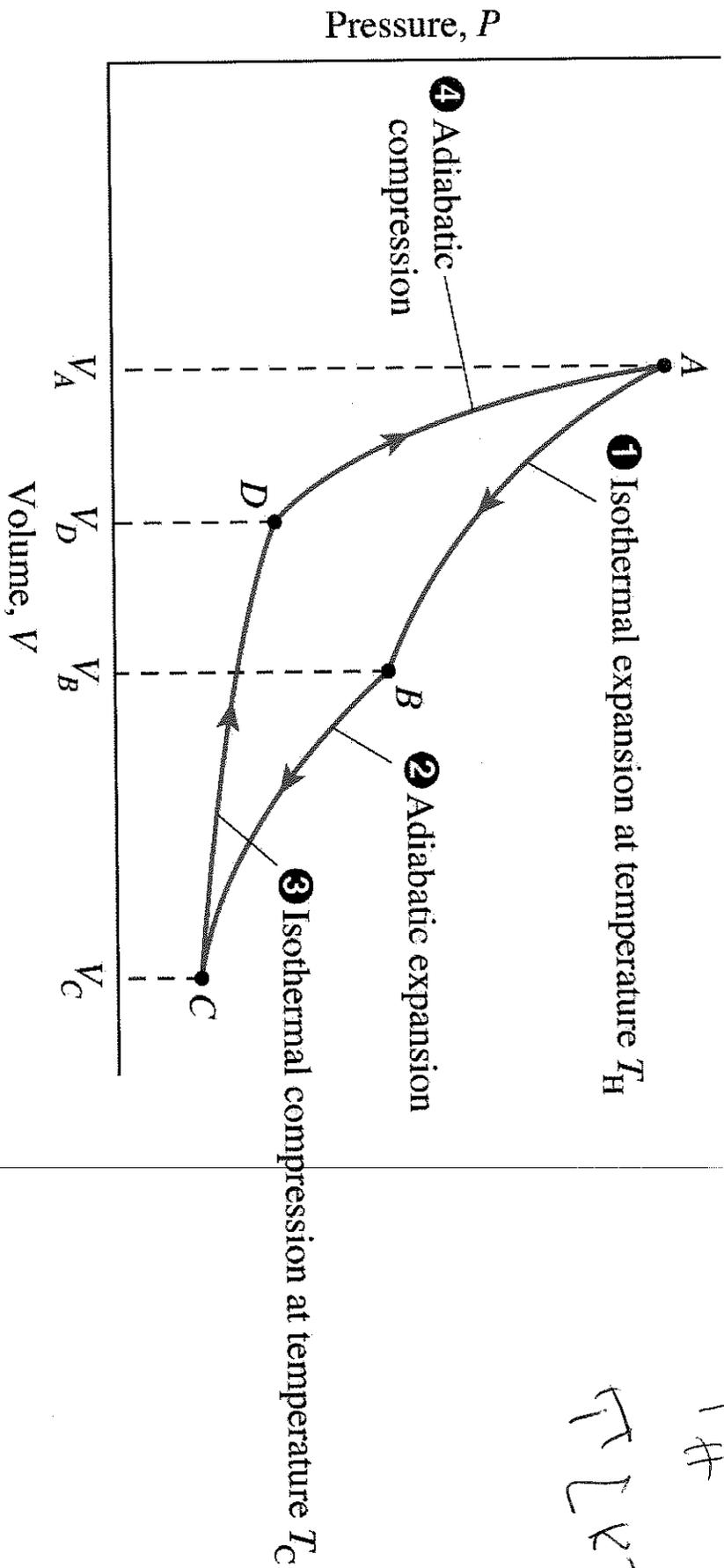


Figure 14.12



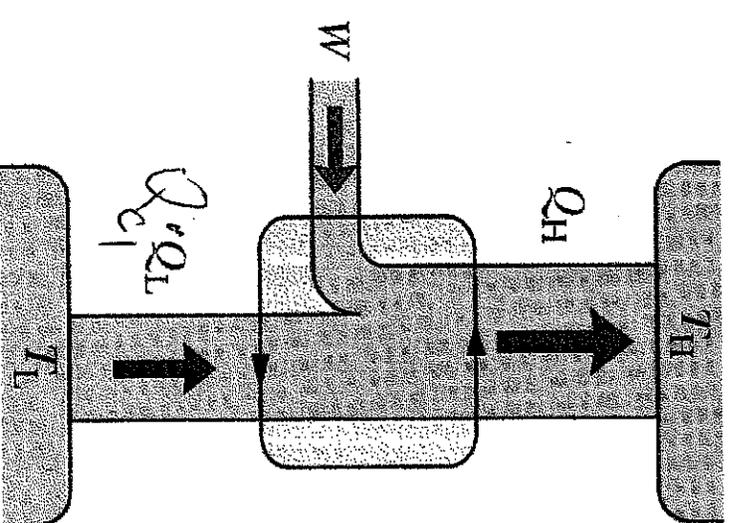
Carnot Engine (How good engines could be?)

$$\eta_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$$

TT [K]

# Refrigerators

- **Refrigerator:** device that uses work to transfer thermal energy from the low-temperature reservoir to the high-temperature reservoir (Fig 20-13)
- **Ideal refrigerator:** processes involved in the refrigerator's operations are reversible



- Coefficient of performance:

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|}$$

$$|W| = |Q_H| - |Q_L|$$

$$K = \frac{|Q_L|}{|Q_H| - |Q_L|}$$

- Carnot (ideal) refrigerator:

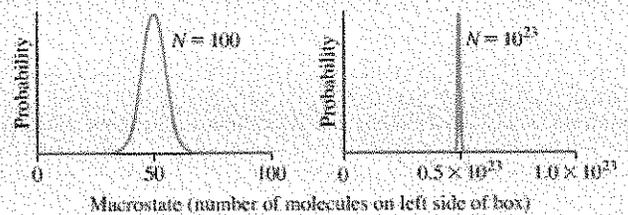
$$\text{COP} = K_C = \frac{T_L}{T_H - T_L} \quad \text{COP} > 1; \quad \text{TEK}$$

### Statistical Interpretation of Entropy

**Section 14.5** Entropy is related to the statistical probability of a system's macroscopic state: the higher the probability, the higher the entropy. A **microstate** is a description of the specific configuration of molecules, noting the position of each. A **macrostate** is a description that does not detail the individual molecules.

**Boltzmann (statistical) entropy:**  $S = k_B \ln \Omega$

More disordered states have higher entropy; more ordered states have lower entropy.



$$200 \text{ Cal} \rightarrow \text{J} \quad | \quad \text{Food C} = 4186 \text{ J}$$

**22. SOLVE** The conversion between food calories to Joules gives:

$$200 \text{ Cal} \left[ \frac{4186 \text{ J}}{1 \text{ Cal}} \right] = 837 \text{ kJ}$$

2.3 mL Hg at 0°C | Q=? to T ↑ 100°C



55. ORGANIZE AND PLAN This is a straightforward use of Equation 13.2:

$Q = mc\Delta T$ , where the specific heat of mercury is from Table 13.1.  $c = 140 \text{ J/kg}\cdot\text{C}$ .

The one thing we will need is the density of liquid mercury from Table 10.1:

$\rho = 13,600 \text{ kg/m}^3$ .

$1 \text{ L} = 10^{-3} \text{ m}^3$

$1 \text{ mL} = 10^{-6} \text{ m}^3$

Known:  $V = 2.30 \text{ mL}$ ,  $\Delta T = 100^\circ\text{C}$ .

SOLVE Plugging the mass of mercury ( $m = \rho V$ ) into Equation 13.2:

$$Q = \rho V c \Delta T = (0.0136 \text{ kg/mL})(2.30 \text{ mL})(140 \text{ J/kg}\cdot\text{C})(100^\circ\text{C}) = 438 \text{ J}$$

REFLECT This heat causes the mercury to expand slightly, which results in the liquid rising inside the thermometer. Because this rise is uniform, we can use it to measure the temperature. In this way, the thermometer works simply by absorbing heat (or losing heat) to the environment.

C 4g

A piece of metal absorbs 3.6 kJ of heat, increasing its temperature by 33 °C. What is its heat capacity ?

By definition  $Q = C\Delta T$  so heat capacity  $C=Q /\Delta T$

In our case  $Q = 3.6 \text{ kJ}$  and  $\Delta T = 33 \text{ }^\circ\text{C}$

Therefore  $C = (3600 \text{ J})/(33 \text{ }^\circ\text{C}) = 109.1 \text{ J/}^\circ\text{C}$

189 L    \$\$\$ = ?     $\Delta T = 60^\circ - 10^\circ \text{C}$  ; \$0.12 kWh

**46. ORGANIZE AND PLAN** We will need Equation 2.2:  $Q = mc\Delta T$ , along with the density of water: 1.00 kg/L. Note too that a kilowatt-hour (kWh) is a unit of energy, so we'll need to convert J into kWh.

*Known:*  $V = 189 \text{ L}$ ,  $R = \$0.12/\text{kWh}$ ,  $\Delta T = 60^\circ\text{C} - 10^\circ\text{C} = 50^\circ\text{C}$ .

**SOLVE** The 189 L is equivalent to 189 kg, so the heat required to raise its temperature by 50 degrees is:

$$Q = mc\Delta T = (189 \text{ kg})(4186 \text{ J/kg}\cdot\text{C})(50^\circ\text{C}) = 3.96 \times 10^7 \text{ J}$$

To convert this to kWh, recall that  $1 \text{ W} = 1 \text{ J/s}$ , or equivalently  $1 \text{ J} = 1 \text{ W}\cdot\text{s}$ :

$$3.96 \times 10^7 \text{ J} \left[ \frac{1 \text{ W}\cdot\text{s}}{1 \text{ J}} \right] \left[ \frac{1 \text{ kW}}{1000 \text{ W}} \right] \left[ \frac{1 \text{ h}}{60 \cdot 60 \text{ s}} \right] = 11 \text{ kWh} \quad (* \$0.12)$$

The cost of this much energy is \$1.32.

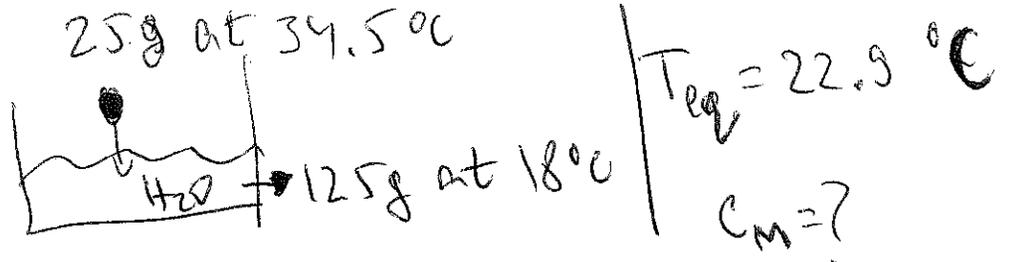
$$P [\text{W}] = \frac{W [\text{J}]}{\Delta t [\text{s}]} ; 1 \text{ W} = \frac{1 \text{ J}}{\text{s}} \rightarrow 1 \text{ J} = 1 \text{ W} \cdot 1 \text{ s}$$

An iron rod of mass 0.5 kg is at temperature of 20 °C. How much heat, Q, in Joules must it absorb so that its temperature raises to 80 °C ?

By definition  $Q = m.c.\Delta T$  where specific heat of iron  $c_{\text{Fe}} = 449 \text{ J}/(\text{kg} \cdot ^\circ\text{C})$

$$\text{Therefore, } Q = (0.5 \text{ kg}) \cdot (449) \cdot (80 - 20) = 13470 \text{ J}$$

Boon



**53. ORGANIZE AND PLAN** We know that the sum of the heat lost by the material and gained by the water is zero:  $Q_M + Q_W = 0$ , so we'll use that to solve for the unknown specific heat:  $c_M$ . The temperature changes for the material and the water are:

$$\Delta T_M = 22.9^\circ\text{C} - 34.5^\circ\text{C} = -11.6^\circ\text{C}$$

$$\Delta T_W = 22.9^\circ\text{C} - 18^\circ\text{C} = 4.9^\circ\text{C}$$

**Known:**  $m_M = 25.0$  g,  $m_W = 125$  g.

**SOLVE** Using the heat exchange and Equation 13.2:

$$c_M = -\frac{m_W c_W \Delta T_W}{m_M \Delta T_M} = -\frac{(0.125 \text{ kg})(4186 \text{ J/kg}\cdot^\circ\text{C})(4.9^\circ\text{C})}{(0.0250 \text{ kg})(-11.6^\circ\text{C})} = 8841 \text{ J/kg}\cdot^\circ\text{C}$$

$$-Q_M = +Q_{H_2O}$$

$$Q = mc\Delta T$$

**REFLECT** Looking through Table 13.1, there's no material that matches this specific heat. But of course this list is not exhaustive, so we shouldn't be concerned.

$$-m_M c_M \Delta T_M = +m_W c_W \Delta T_W$$

$$c_M = \frac{m_W c_W \Delta T_W}{m_M \Delta T_M}$$

$$c_W = 4186 \text{ J/kg}\cdot^\circ\text{C}$$

↓  
Table 13.1

56 g  $N_2$  at  $25^\circ\text{C}$  + 12 g He at  $45^\circ\text{C}$

$T_{eq} = ?$

**57. ORGANIZE AND PLAN** The nitrogen starts off colder, so it will gain heat from the helium: ( $Q_N = -Q_{He}$ ). We'll assume that the gases are combined under fixed pressure, so that the heat gained or lost will come from Equation 8.4:  $Q = nc_p\Delta T$ . The molar specific heats can be taken from Table 13.2 for nitrogen ( $c_{N_2} = 29.1 \text{ J/mol}\cdot^\circ\text{C}$ ) and for helium ( $c_{He} = 20.8 \text{ J/mol}\cdot^\circ\text{C}$ ). We'll need to convert the given masses into moles, and write the temperature change for the nitrogen as:  $\Delta T_N = T_f - 25^\circ\text{C}$ , and the helium as:  $\Delta T_{He} = T_f - 45^\circ\text{C}$ .

Known:  $m_N = 56 \text{ g}$ ,  $m_{He} = 12 \text{ g}$

**SOLVE** The equal but opposite heat exchange implies:

$$-Q_N = +Q_{He} \Rightarrow n_N c_N \Delta T_N = -n_{He} c_{He} \Delta T_{He}$$

The molar masses are 28 g/mol for nitrogen gas and 4 g/mol for helium gas, so the number of moles are 2 mol of nitrogen and 3 mol of helium. Solving for the final equilibrium temperature:

$$\frac{\Delta T_{N_2}}{T_f - 25^\circ\text{C}} = - \frac{(3 \text{ mol})(20.8 \text{ J/mol}\cdot^\circ\text{C}) \Delta T_{He}}{(2 \text{ mol})(29.1 \text{ J/mol}\cdot^\circ\text{C}) (T_f - 45^\circ\text{C})} \Rightarrow T_f = \underline{\underline{35^\circ\text{C}}}$$

56g  $N_2 = n \cdot 28 \frac{\text{g}}{\text{mol}}$   
 12g He =  $n \cdot 4 \frac{\text{g}}{\text{mol}}$

$n_{N_2} = 2$

$n_{He} = 3$

**REFLECT** The answer makes sense, since the final temperature is halfway between the initial temperatures of the nitrogen and the helium. If you assumed that the gases were mixed with constant volume, the result would be practically the same:  $T_f = 34^\circ\text{C}$ . This is because the ratio of the molar specific heats ( $c_{He}/c_N$ ) is practically the same for constant volume and constant pressure.

Table 13.2 | He  $\rightarrow c_p = 20.8 \text{ mol}\cdot^\circ\text{C}$   
 $N_2 \rightarrow c_p = 29.1 \text{ mol}\cdot^\circ\text{C}$

How much heat,  $Q$ , is required to melt 500 g of ice at  $0^\circ\text{C}$  ?

By definition  $Q = mL_f$  where the heat of fusion of ice is  $L_{\text{ice}} = 3.33 \times 10^5 \text{ J/kg}$

$$\text{Therefore, } Q = (0.5 \text{ kg}) \cdot (3.33 \times 10^5) = 166.5 \text{ kJ}$$

45. You mix 18 kg of water at 25 °C with 6 kg of water at 2 °C, what is the final temperature ?

All we can say is that the hotter water changes temperature by:  $\Delta T_{\text{hot}} = T_f - 25^\circ\text{C}$ , while the colder water changes temperature by:  $\Delta T_{\text{cold}} = T_f - 2.0^\circ\text{C}$ . We will be able to solve for  $T_f$  using Equation 13.2,

$$\text{i.e. } Q = c.m. \Delta T$$

and the fact that the heat lost by the hot water is gained by the cold water:  $Q_{\text{hot}} = -Q_{\text{cold}}$ , assuming of course that no heat is lost to the surroundings.

*Known:*  $m_{\text{hot}} = 18 \text{ kg}$ ,  $m_{\text{cold}} = 6 \text{ kg}$ . and  $c_{\text{water}} = 4186 \text{ J}/(\text{kg} \cdot ^\circ\text{C})$  (Table 13.1)

**SOLVE** The equal but opposite heat exchange implies:

$$Q_{\text{hot}} = -Q_{\text{cold}} \Rightarrow m_{\text{hot}} c \Delta T_{\text{hot}} = -m_{\text{cold}} c \Delta T_{\text{cold}}$$

Solving for the final temperature:

$$T_f - 25^\circ\text{C} = -\frac{6 \text{ kg}}{18 \text{ kg}}(T_f - 2.0^\circ\text{C}) \Rightarrow T_f = 19^\circ\text{C}$$

# Sample Problem 18-3

(a) How much heat must be absorbed by ice of mass  $m = 720$  g at  $T_1 = -10^\circ\text{C}$  to take it to liquid state at  $T_3 = 15^\circ\text{C}$ ?

Let  $T_2 = 0^\circ\text{C}$ . Then

$$Q_{12} = c_{\text{ice}}m(T_2 - T_1) = (2,220 \text{ J/kg K})(0.72 \text{ kg})[0^\circ\text{C} - (-10^\circ\text{C})] \\ = 15,984 \text{ J} = 15.98 \text{ kJ} \quad \xrightarrow{\text{Table 13.3}}$$

$$Q_{\text{F}} = L_{\text{F}} m = (333 \text{ kJ/kg})(0.720 \text{ kg}) = 239.8 \text{ kJ}$$

$$Q_{23} = c_{\text{w}} m (T_3 - T_2) = (4,190 \text{ J/kg K})(0.720 \text{ kg})(15^\circ\text{C} - 0^\circ\text{C}) \\ = 45,252 \text{ J} = 45.25 \text{ kJ}$$

$$Q = Q_{12} + Q_{\text{F}} + Q_{23} = 15.98 \text{ kJ} + 239.8 \text{ kJ} + 45.25 \text{ kJ} = 300 \text{ kJ}$$

You have 300 g of coffee at 55 °C. How much 10 °C water do you need to add in order to reduce the coffee's temperature to a more bearable 49 °C? Note the specific heat,  $c$ , of coffee and water are the same ( $c = 4186 \text{ J} \cdot \text{kg}^{-1} \cdot \text{C}^{-1}$ )

Heat is exchanged between the hot coffee and the cold water, but the whole system does not lose or receive heat. Therefore,  $Q_{\text{hot}} + Q_{\text{cold}} = 0$ . The temperature of the coffee drops, while that of the added water rises:

$$\Delta T_{\text{hot}} = 49^\circ\text{C} - 55^\circ\text{C} = -6^\circ\text{C}$$

$$\Delta T_{\text{cold}} = 49^\circ\text{C} - 10^\circ\text{C} = 39^\circ\text{C}$$

$$\textit{Known: } m_{\text{hot}} = 300 \text{ g}$$

**SOLVE** The heat exchange is written:

$$Q_{\text{hot}} + Q_{\text{cold}} = m_{\text{hot}}c\Delta T_{\text{hot}} + m_{\text{cold}}c\Delta T_{\text{cold}} = 0$$

The specific heat of coffee is the same as water, so the  $c$ 's will cancel out of the equation. Solving for the cold water mass:

$$m_{\text{cold}} = -m_{\text{hot}} \frac{\Delta T_{\text{hot}}}{\Delta T_{\text{cold}}} = -(300 \text{ g}) \frac{(-6^\circ\text{C})}{(39^\circ\text{C})} = 46 \text{ g}$$

# Sample Problem 18-3 (cont)

(b) If we supply the ice with a total energy of only 210 kJ (as heat), what then are the final state and the temperature of the water?

$$Q_{12} = 15.98 \text{ kJ}, Q_F = 239.8 \text{ kJ}, Q_{23} = 45.25 \text{ kJ}$$

Final state: ICE and WATER,  $T_f = 0^\circ \text{C}$

$$Q_{\text{rem}} = 210 \text{ kJ} - 15.98 \text{ kJ} = 194 \text{ kJ}$$

$$Q_{\text{rem}} = m_w L_F$$

$$m_w = Q_{\text{rem}} / L_F = 194 \text{ kJ} / 333 \text{ kJ/kg} = 0.583 \text{ kg}$$

$$m_{\text{ice}} = 0.720 \text{ kg} - 0.583 \text{ kg} = 0.137 \text{ kg}$$

P 25. Calculate the amount of energy, in joules, required to completely melt 130 g silver initially at 15 C°.

$$T_R \approx T^{\circ}\text{C} + 273.15$$

- 25. The melting point of silver is 1235 K, so the temperature of the silver must first be raised from 15.0° C (= 288 K) to 1235 K. This requires heat

$$Q = cm(T_f - T_i) = (236 \text{ J/kg} \cdot \text{K})(0.130 \text{ kg})(1235^{\circ}\text{K} - 288^{\circ}\text{K}) = 2.91 \times 10^4 \text{ J.}$$

- Now the silver at its melting point must be melted. If  $L_F$  is the heat of fusion for silver this requires

$$Q = mL_F = (0.130 \text{ kg})(105 \times 10^3 \text{ J/kg}) = 1.36 \times 10^4 \text{ J.}$$

- The total heat required is (  $2.91 \times 10^4 \text{ J} + 1.36 \times 10^4 \text{ J} ) = 4.27 \times 10^4 \text{ J.}$

A window with area of  $2 \text{ m}^2$  is made of 4 mm thick glass. If it is  $20 \text{ }^\circ\text{C}$  colder outside than inside, what is the heat flow rate,  $H$ , through the window ?

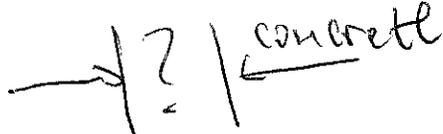
By definition  $H = k \cdot A \cdot (\Delta T / \Delta x)$

In our case thermal conductivity of glass  $k = 0.8 \text{ W}/(\text{C}^\circ \cdot \text{m})$ ,  $A = 2 \text{ m}^2$ ,  $\Delta T = 20 \text{ }^\circ\text{C}$   
and  $\Delta x = 0.004 \text{ m}$

---

Therefore,  $H = (0.8) \cdot (2) \cdot (20) / (0.004) = 8000 \text{ W}$

Boon



$$\text{So } R_c = R_{\text{wood}} (1.8 \text{ cm})$$

**94. ORGANIZE AND PLAN** We are asked to compare the insulation provided by two materials. In Problems 13.78 through 13.81, we were introduced to the  $R$ -value of a material, which is defined as the thickness divided by the thermal conductivity:  $R = \Delta x / k$ . To find the  $R$ -values for concrete and wood walls, we'll need their respective thermal conductivities from Table 13.4:

$$k_c = 1.28 \text{ W/}^\circ\text{C}\cdot\text{m} \text{ and } k_w = 0.12 \text{ W/}^\circ\text{C}\cdot\text{m}.$$

**KNOWN:**  $\Delta x_w = 1.8 \text{ cm}$ .

$$\frac{\Delta x_c}{k_c} = \frac{\Delta x_{\text{wood}}}{k_{\text{wood}}} = R$$

**SOLVE** For a concrete wall to have the same  $R$ -value of a piece of wood, its thickness would need to be:

$$\Delta x_c = \Delta x_w \frac{k_c}{k_w} = (1.8 \text{ cm}) \frac{(1.28 \text{ W/}^\circ\text{C}\cdot\text{m})}{(0.12 \text{ W/}^\circ\text{C}\cdot\text{m})} = 19.2 \text{ cm}$$

**REFLECT** To get the same insulation as a piece of wood, the thickness of a concrete wall has to be over ten times thicker. That's because concrete lets heat escape faster than wood does. Think of a cold concrete floor vs. a warm wooden one.

$$\cancel{k_c} A \frac{\Delta T}{\Delta x_c} = H = \cancel{k_w} A \frac{\Delta T}{\Delta x_w}$$

$$\frac{k_c}{\Delta x_c} = \frac{k_w}{\Delta x_w}$$

$$\Delta x_c = \Delta x_w \frac{k_c}{k_w}$$

A sphere of surface area  $2 \text{ m}^2$  and emissivity of 0.5 is at temperature of  $300 \text{ }^\circ\text{C}$ .  
What is the rate at which the sphere radiates heat into empty space ?

The rate,  $P$ , at which an object at temperature  $T$  radiates energy is:

$$P = e \cdot \sigma \cdot A \cdot T^4, \text{ where } \sigma = 5.67 \times 10^{-8} \text{ W}/(\text{m}^2 \cdot \text{K}^2)$$

---

$$\text{In our case } e = 0.5, A = 2 \text{ m}^2 \text{ and } T = 300 + 273.15 = 573.15 \text{ K}$$

Therefore,

$$P = (0.5) \cdot (5.67 \times 10^{-8}) \cdot (2) \cdot (573.15)^4 = 6118 \text{ W}$$

Boone



$$R_{\text{Sun}} = 6.96 \times 10^8 \text{ m}$$

$$T_{\text{Sun}} = 5800 \text{ K}$$

**82. ORGANIZE AND PLAN** The Stefan-Boltzmann law (Equation 13.8) tells us the rate at which a body radiates energy:  $P = e\sigma AT^4$ . Saying the Sun is a blackbody means that its emissivity is one, i.e.:  $e = 1$ . The Stefan-Boltzmann constant is:  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$ , and the surface area of a sphere is:  $A = 4\pi r^2$ .

*Known:*  $r = 6.96 \times 10^8 \text{ m}$ ,  $T = 5800 \text{ K}$ .

**SOLVE** Substituting the values into the Stefan-Boltzmann law:

$$P = e\sigma AT^4 = (1)(5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4)(4\pi(6.96 \times 10^8 \text{ m})^2)(5800 \text{ K})^4 = 3.91 \times 10^{26} \text{ W}$$

**REFLECT** Currently, the world uses somewhere around 15 TW ( $1.5 \times 10^{13} \text{ W}$ ) of power. In comparison, the Sun emits over 10 quadrillion times the energy we use.

$$1.25 \text{ L He} \rightarrow \Delta T = 100^\circ\text{C}; \Delta U = ?$$

**27. ORGANIZE AND PLAN** Helium is a monatomic gas so the internal energy difference when warmed is  $\Delta U = \frac{3}{2} N k_B \Delta T$ . The number of atoms in 1.25 L is given by the ideal gas law.

*Known:*  $V = 1.25 \text{ L}; \Delta T = 100^\circ\text{C}$ .

**SOLVE** From the ideal gas law  $PV = N k_B T$  we find that internal energy increases by:

$$\Delta U = \frac{3}{2} N k_B \Delta T = \frac{3}{2} P V \frac{\Delta T}{T}$$

$$\rightarrow N k_B = \frac{PV}{T}$$

$$1 \text{ L} = 10^{-3} \text{ m}^3$$

Insert known values:

$$\Delta U = \frac{3}{2} (1 \text{ atm})(1.25 \text{ L}) \frac{(100^\circ\text{C})}{(20^\circ\text{C})} = \frac{3}{2} (101 \text{ kPa})(1.25 \times 10^{-3} \text{ m}^3) \frac{(373 \text{ K})}{(293 \text{ K})} = 64.8 \text{ J}$$

**REFLECT** We will assume room temperature and atmospheric pressure unless stated otherwise (or unless such an assumption appears obviously erroneous). Note that it's important to convert the room temperature to Kelvin because it's not a temperature difference.

$$\uparrow 45 \text{ W} = \frac{\Delta U}{\Delta t} \leftarrow \Delta U \uparrow \quad \text{W rate } 165 \text{ W} \quad \left| \quad \frac{\Delta Q}{\Delta t} = ?$$

**29. ORGANIZE AND PLAN** Heat, internal energy, and work are related through the first law of thermodynamics. We will divide the quantities in this law by time to get powers and heat flow rates.

*Known:*  $\Delta U/\Delta t = 45 \text{ W}$ ;  $P = -165 \text{ W}$ .

**SOLVE** The first law of thermodynamics divided by time is:

$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t} + P$$

$$\leftarrow \Delta U = \Delta Q + W / \Delta t$$

which we can rewrite to get an expression for the heat flow into the system:

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta U}{\Delta t} - P = (45 \text{ W}) - (-165 \text{ W}) = 210 \text{ W}$$

and

$$\frac{W}{\Delta t} = P$$

**REFLECT** The power is negative because the system is doing work.

$$P [W]$$

$$[W] = \frac{J}{s}$$

# Sample Problem 18-5

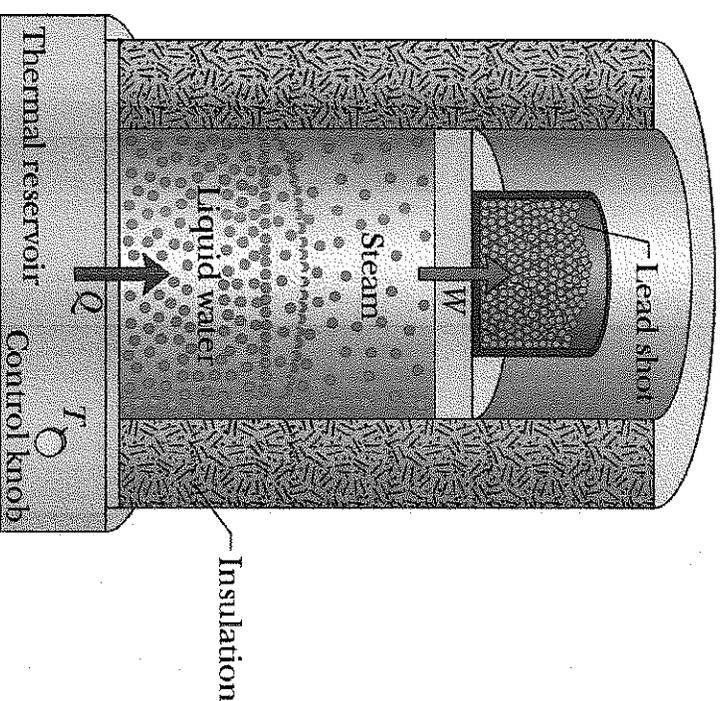
Let 1.0 kg of liquid water at 100°C be converted to steam at 100°C by boiling at standard atmospheric pressure (which is 1.0 atm or  $1.01 \times 10^5$  Pa) in the arrangement of the figure. The volume of the water changes from an initial value of  $1.0 \times 10^{-3}$  m<sup>3</sup> as a liquid to  $1.672$  m<sup>3</sup> as steam.

(a) How much work is done by the system?

$$W = -p(V_f - V_i)$$

$$= -(1.01 \times 10^5 \text{ Pa})(1.67 \text{ m}^3 - 1.0 \times 10^{-3} \text{ m}^3)$$

$$= -1.69 \times 10^5 \text{ J} = -169 \text{ kJ}$$



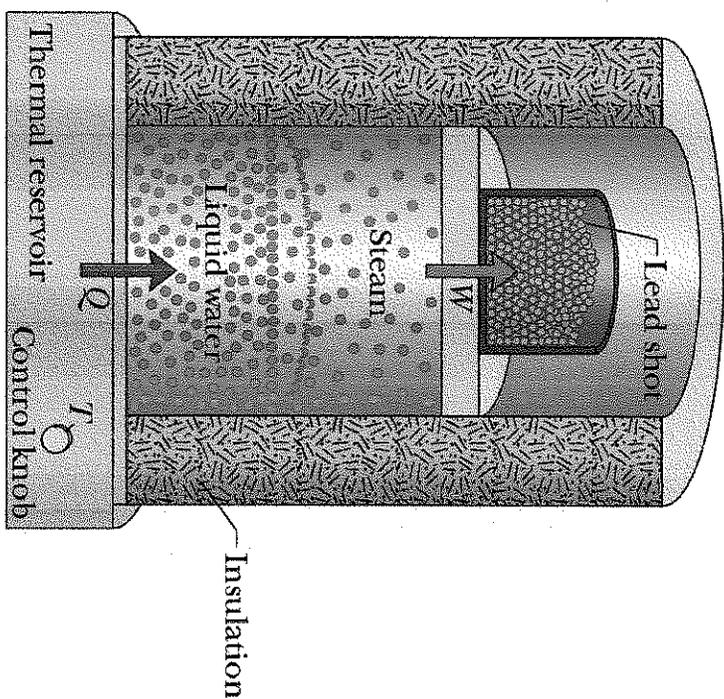
# Sample Problem 18-5 (cont)

(b) How much energy is transferred as heat during the process? *to the system*

$$Q = L_V m = (2256 \text{ kJ/kg})(1.0 \text{ kg}) = 2,256 \text{ kJ}$$

(c) What is the change in the system's internal energy?

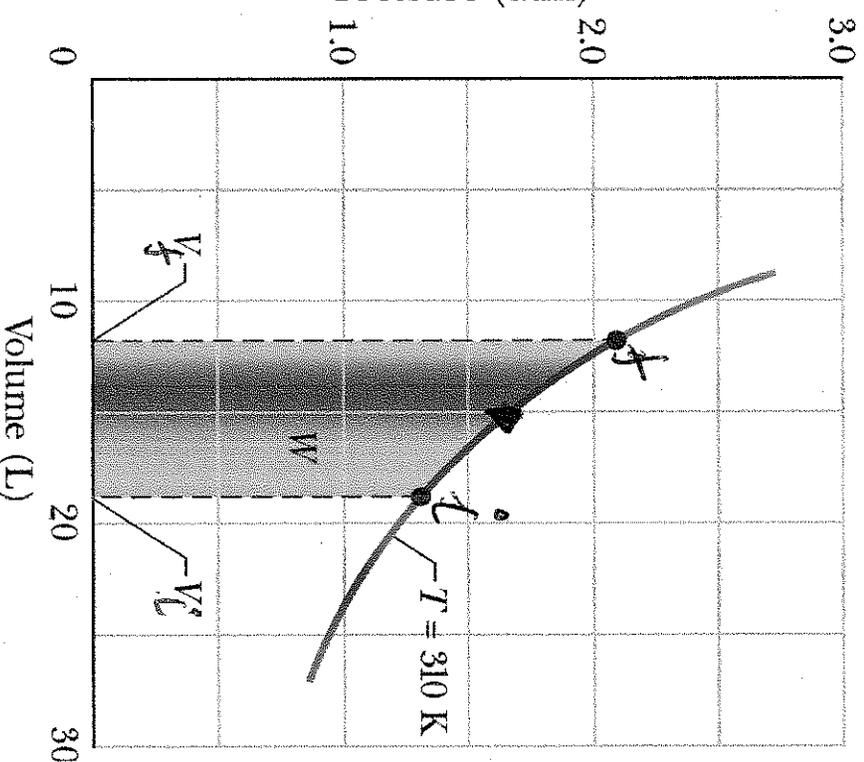
$$\Delta U = Q + W = 2256 \text{ kJ} - 169 \text{ kJ} = 2,087 \text{ kJ}$$



# Sample Problem 19-2

One mole of oxygen (assume to be an ideal gas) expands at a constant temperature  $T$  of 310 K from an initial volume  $V_i$  of 12 L to a final volume  $V_f$  of 19 L. How much work is done on the gas during the expansion?

$$\begin{aligned}
 W &= -nRT \ln \frac{V_f}{V_i} \\
 &= nRT \ln \frac{V_i}{V_f} \\
 &= (1 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(310 \text{ K}) \ln \left( \frac{12 \text{ L}}{19 \text{ L}} \right) \\
 &= 1183 \text{ J}
 \end{aligned}$$



0.3 mol He 

$$V_f = 5V_i \text{ at } 300 \text{ K} \quad | \quad W_{\text{gas}} = ?$$

**38. ORGANIZE AND PLAN** We will use the formula for work done on the gas in an isothermal process.

*Known:*  $n = 0.30 \text{ mol}$ ;  $T = 300 \text{ K}$ ;  $V_f = 5V_i$ .

**SOLVE** The work done on the gas is:

$$W = nRT \ln \left( \frac{V_i}{V_f} \right) = (0.30 \text{ mol})(8.31 \text{ J/(mol} \cdot \text{K)})(300 \text{ K}) \ln \left( \frac{1}{5} \right) = -1.2 \text{ kJ}$$

The work done by the gas is positive 1.2 kJ.

**REFLECT** It does not matter in an isothermal process whether the gas is monatomic or diatomic.

0.1 mol gas at 300K,  $\nabla P$  at  $T = \text{const}$   
 $Q = ?$  so  $V_f = 3V_i$

**42. ORGANIZE AND PLAN** The work done on an expanding gas is negative. The sum of the work done to and the heat added to an isothermal system is zero. To calculate a precise value we use the formula for work done on an isothermal system.

*Known:*  $n = 0.10 \text{ mol}$ ;  $T = 300 \text{ K}$ ;  $V_f = 3V_i$ .

**SOLVE** (a) When the gas is expanding the work done on the gas is negative, so the heat added is positive, i.e., heat flows into the gas.

(b) The work done on an isothermal system is:

$$W = nRT \ln\left(\frac{V_i}{V_f}\right)$$

$0 = \Delta U \sim E_{th} = Q + W$   
 $W \text{ is } (-W) \text{ so } Q > 0$   
for  $T = \text{const}$

and  $W + Q = 0$ , so the required heat is:

$Q = -W$

$$Q = -W = -nRT \ln\left(\frac{V_i}{V_f}\right) = -(0.10 \text{ mol})(8.31 \text{ J/(mol} \cdot \text{K)})(300 \text{ K}) \ln\left(\frac{1}{3}\right) = 0.27 \text{ kJ}$$

**REFLECT** It is straightforward to see that the answer is reasonable, because  $\ln(3) \approx 1$ .

Expansion  $Q=0$ , 2 moles  $H_2$  do 750 J of work

$\Delta T = ?$

**46. ORGANIZE AND PLAN** The work done on the gas in an adiabatic process is proportional to the difference between two products: the final pressure times volume minus the initial pressure times volume. From the ideal gas law this means that the work done on the gas is proportional to the difference between final and initial temperature.

*Known:*  $n = 2.0 \text{ mol}$ ;  $W = -750 \text{ J}$ .

**SOLVE** (a) The gas does positive work. This means the work done on the gas is negative, and from our argument above that means the final temperature is less than the initial temperature, so the gas temperature decreases.

The reason for this is that to do work the gas must expend its internal energy, which is proportional to temperature for a diatomic gas.

(b) The work done in an adiabatic process is:

$$W = \frac{P_f V_f - P_i V_i}{\gamma - 1}$$

$\gamma = \frac{7}{2}$  for diatomic

Rewrite this expression using the ideal gas law:

$$W = \frac{P_f V_f - P_i V_i}{\gamma - 1} = \frac{nR}{\gamma - 1} (T_f - T_i) = \frac{nR\Delta T}{\gamma - 1}$$

$P_f V_f = nRT_f$   
 $P_i V_i = nRT_i$

Solve for the temperature difference and insert know values:

$$\Delta T = \frac{(\gamma - 1)W}{nR} = \frac{\left(\frac{7}{2} - 1\right)(-750 \text{ J})}{(2.0 \text{ mol})(8.31 \text{ J/(mol} \cdot \text{K)})} = -18 \text{ K}$$

$P_f V_f - P_i V_i = nR(T_f - T_i)$

**REFLECT** The expression we derived here between the work done and the temperature difference holds true for all adiabatic processes if the gas is an ideal gas.

Example: A flexible container contains  $2.42 \times 10^{-5} \text{ m}^3$  of fluid at room T. Somebody pushes on the container, maintaining a constant 1-atm pressure, and reduces its volume by 25 %. How much work is done on the fluid ?

$$\begin{aligned} \text{Work done on the fluid: } W &= -P(V_f - V_i) = 1\text{atm}(0.75V_i - V_i) = \\ &= - (1.013 \times 10^5 \text{ Pa})V_i(-0.25) = (1.013 \times 10^5 \text{ Pa})(2.42 \times 10^{-5} \text{ m}^3)(0.25) = 0.61 \text{ J} \end{aligned}$$

100g ice  $\rightarrow$  H<sub>2</sub>O at 0°C; 100g H<sub>2</sub>O  $\xrightarrow{100^\circ\text{C}}$  steam

57. **ORGANIZE AND PLAN** The entropy change is the heat added to the ice or the water divided by the temperature. The heat added can be calculated using the heats of transformation from Table 13.3.

*Known:*  $m = 100 \text{ g}$ ;  $T_f = 0^\circ\text{C}$ ;  $L_f = 3.33 \times 10^5 \text{ J/kg}$ ;  $T_v = 100^\circ\text{C}$ ;  $L_v = 2.26 \times 10^6 \text{ J/kg}$ .

**SOLVE** When the ice melts the entropy change is:

$$\Delta S_f = \frac{Q_f}{T_f} = \frac{mL_f}{T_f} = \frac{(100 \text{ g})(3.33 \times 10^5 \text{ J/kg})}{(0^\circ\text{C})} = 122 \text{ J/K}$$

When the water boils the entropy change is:

$$\Delta S_v = \frac{Q_v}{T_v} = \frac{mL_v}{T_v} = \frac{(100 \text{ g})(2.26 \times 10^6 \text{ J/kg})}{(100^\circ\text{C})} = 606 \text{ J/K}$$

Problem # 2 How much energy must be transferred as heat for a reversible isothermal expansion of an ideal gas at  $132^{\circ}\text{C}$  if the entropy of the gas increases by  $46\text{ J/K}$  ?

- From Eq. 20-2,  $\Delta S = S_f - S_i = \frac{Q}{T}$

we obtain

$$Q = T\Delta S = (405\text{ K})(46.0\text{ J/K}) = 1.86 \times 10^4\text{ J.}$$

$\Delta S = ?$  when 75g steam  $\xrightarrow{100^\circ\text{C}}$   $\text{H}_2\text{O}$

**56. ORGANIZE AND PLAN** The entropy change is the heat removed from the steam divided by the temperature. The heat removed can be calculated using the heat of vaporization from Table 13.3.

Bev

*Known:*  $m = 75 \text{ g}$ ,  $T = 100^\circ\text{C}$ ,  $L_v = 2.26 \times 10^6 \text{ J/kg}$

**SOLVE** The heat removed from the steam to condense it is:

$$Q = -mL_v = -(75 \text{ g})(2.26 \times 10^6 \text{ J/kg}) = -1.7 \times 10^5 \text{ J}$$

The entropy change is:

0.515 J/K \* Table 13.3

$$\Delta S = \frac{Q}{T} = \frac{(-1.7 \times 10^5 \text{ J})}{(100^\circ\text{C})} = \frac{(-1.7 \times 10^5 \text{ J})}{(373 \text{ K})} = -4.5 \times 10^2 \text{ J/K}$$

However  $\Delta S$  in the Universe. Since, for example, refrigerator does work to remove the heat and dump it in the Universe

$$W = 650 \text{ J} \rightarrow Q_L = 1270 \text{ J}; \quad \epsilon = ?$$

63. . **ORGANIZE AND PLAN** The efficiency is the ratio between work done and heat used.

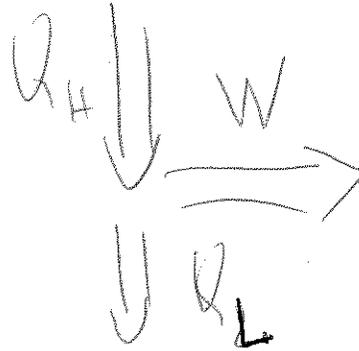
*Known:*  $W = 650 \text{ J}$ ;  $Q_H = 1270 \text{ J}$ .

**SOLVE** The heat engine's efficiency is:

$$e = \frac{W}{Q_H} = \frac{(650 \text{ J})}{1920 \text{ J}} = 0.338 \sim 34\%$$

$$Q_H = W + Q_L \\ = 1920 \text{ J}$$

**P-**



# Sample Problem 20-4

Imagine a Carnot engine that operates between the temperatures  $T_H = 850\text{ K}$  and  $T_L = 300\text{ K}$ . The engine performs  $1200\text{ J}$  of work each cycle, which takes  $0.25\text{ s}$ .

(a) What is the efficiency of this engine?

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{300\text{ K}}{850\text{ K}} = 0.647$$

(b) What is the average power of this engine?

$$P = \frac{W}{t} = \frac{1200\text{ J}}{0.25\text{ s}} = 4800\text{ W} = 4.8\text{ kW}$$

Jet engine ;  $e = ?$

**96. ORGANIZE AND PLAN** The maximum efficiency is that of a Carnot engine, one minus the temperature ratio between the cold and hot reservoirs.

*Known:*  $T_H = 1050^\circ\text{C}$ ;  $T_C = 590^\circ\text{C}$ .

**SOLVE** The maximum efficiency is:

$$e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{(590^\circ\text{C})}{(1050^\circ\text{C})} = 0.348 = 1 - \frac{(590 + 273)}{(1050 + 273)}$$

**REFLECT** If you could invent a way of operating a jet engine such that the exhaust temperature is that of the surrounding air (typically  $-40^\circ\text{C}$  at the cruising height of commercial airliners), you would raise the maximum efficiency to better than 0.8!

$$T_H = ? \text{ if } T_C = 20^\circ\text{C}$$

68. **ORGANIZE AND PLAN** In a Carnot cycle, the efficiency is one minus the temperature ratio between the cold and hot reservoirs.

*Known:*  $T_C = 20^\circ\text{C}$ ;  $e_{\text{Carnot}} = 0.5$ .

**SOLVE** The Carnot efficiency is:

$$e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$$

$$\rightarrow \frac{T_C}{T_H} = 1 - e \rightarrow T_H = \frac{T_C}{1 - e}$$

which we can rewrite for to calculate the maximum temperature:

$$T_H = \frac{1}{1 - e_{\text{Carnot}}} T_C = \frac{1}{1 - 0.5} 20^\circ\text{C} = 40^\circ\text{C} \rightarrow 293\text{K} \approx (20 + 273) \approx 600\text{K}$$

**REFLECT** When the Carnot efficiency is 0.5, the hot reservoir is twice the temperature of the cold reservoir.

(in K)

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|}$$

$$|W| = |Q_H| - |Q_L|$$

$$K = \frac{|Q_L|}{|Q_H| - |Q_L|}$$

## Problem

A Carnot refrigerator does 200 J of work to remove 600 J from its cold compartment. (a) What is the refrigerator's coefficient of performance? (b) How much energy per cycle is exhausted to the kitchen as heat?

34. (a) We use Eq. 21-12,

$$\frac{Q_C}{Q_H - Q_C} = K = \frac{|Q_L|}{|W|} = \frac{600}{200} = 3.$$

(b) Energy conservation for a refrigeration cycle requires  $|Q_L| + |W| = |Q_H|$ , so that the result is 800 J.

$$Q_H - Q_C = W$$

$$Q_H = W + Q_C = 200 \text{ J} + 600 \text{ J} = 800 \text{ J}$$

$$T_H = 310^\circ\text{C} \quad \left| \quad \text{winter } (T_C = 0^\circ\text{C}) \quad P = 650 \text{ MW} \right.$$

$$e_{\text{winter}} = ? \quad ; \quad P_{\text{summer}} = ? \quad \text{when } T_C = 38^\circ\text{C}$$

**89. ORGANIZE AND PLAN** We will assume that the power plant operates at the maximum thermodynamic efficiency, i.e., the efficiency of a Carnot cycle, where the efficiency is one minus the temperature ratio between the cold and hot reservoirs. The power produced is proportional to the efficiency, so from knowing the winter production and both the winter and summer efficiencies we can calculate the summer production.

*Known:*  $T_H = 310^\circ\text{C}$ ;  $P_{\text{winter}} = 650 \text{ MW}$ ;  $T_{C,\text{winter}} = 0^\circ\text{C}$ ;  $T_{C,\text{summer}} = 38^\circ\text{C}$ .

**SOLVE** (a) The theoretical maximum winter efficiency is:

$$e_{\text{Carnot,winter}} = 1 - \frac{T_{C,\text{winter}}}{T_H} = 1 - \frac{(0^\circ\text{C})}{(310^\circ\text{C})} = 0.532 \quad \rightarrow + 273$$

(b) The theoretical maximum summer efficiency is:

$$e_{\text{Carnot,summer}} = 1 - \frac{T_{C,\text{summer}}}{T_H} = 1 - \frac{(38^\circ\text{C})}{(310^\circ\text{C})} = 0.466 \quad \rightarrow + 273$$

The production is proportional to efficiency, so comparing the winter and summer numbers we must have:

$$1222 \text{ MW} = \frac{P_{\text{winter}}}{e_{\text{winter}}} = \frac{P_{\text{summer}}}{e_{\text{summer}}}$$

If we assume that the efficiencies are the Carnot efficiencies, we have:

$$\frac{P_{\text{winter}}}{e_{\text{Carnot,winter}}} = \frac{P_{\text{summer}}}{e_{\text{Carnot,summer}}}$$

$$P_{\text{summer}} = \frac{e_{\text{Carnot,summer}}}{e_{\text{Carnot,winter}}} P_{\text{winter}} = \frac{(0.466)}{(0.532)} (650 \text{ MW}) = 570 \text{ MW}$$

**Problem** A Carnot engine has an efficiency of 22%. It operates between constant-temperature reservoirs differing in temperature by 75 K. What is the temperature of the (a) lower-temperature and (b) higher-temperature reservoir?

25. We solve (b) first

$$\epsilon = 1 - \frac{T_L}{T_H}$$

$$\text{Carnot cycle efficiency} = \frac{T_H - T_L}{T_H} = \epsilon$$

(b) For a Carnot engine, the efficiency is related to the reservoir temperatures by Eq. 20.17. Therefore,

$$\epsilon = \frac{T_H - T_L}{T_H} = \frac{75 \text{ K}}{T_H} = 0.22$$

which is equivalent to 68°C.  $T = 273 + 68 = 341 \text{ K}$

(a) The temperature of the cold reservoir is  $T_L = 341 - 75 = 266 \text{ K}$

$$\Delta T = 75 \text{ K} = 341 \text{ K} - T_L$$

$$\frac{Q_c}{W} = \text{COP} = 3.8 ; P = 600 \text{ W} = \frac{W}{t} \quad \left| \quad \frac{Q_c}{t} = ? \right.$$

**71. ORGANIZE AND PLAN** The coefficient of performance of a refrigerator is the heat removed divided by the required work.

*Known:* COP = 3.8;  $P = 600 \text{ W}$ .

**SOLVE** The refrigerator can remove heat at a rate:

$$\frac{Q_c}{t} = \text{COP} \times \frac{W}{t} \rightarrow \frac{Q_c}{t} = \text{COP} \frac{W}{t} = \text{COP} \times P = (3.8)(600 \text{ W}) = 2.3 \text{ kJ/s}$$

**REFLECT** The higher the COP the larger the amount of heat removed.

per 1 sec  $Q_H = 1700 \text{ MJ}; Q_C = 1100 \text{ MJ} \quad \left| \begin{array}{l} P = ? \\ e = ? \end{array} \right.$

- 65. ORGANIZE AND PLAN** The efficiency is the ratio between work done and the thermal energy removed from the uranium fuel. The work done is the difference between the thermal energy and the waste heat.

*Known:*  $Q_H = 1700 \text{ MJ}; Q_C = 1100 \text{ MJ}; t = 1 \text{ s}.$

**SOLVE** (a) The work done is:

$$Q_H = W + Q_C$$

$$W = Q_H - Q_C = (1700 \text{ MJ}) - (1100 \text{ MJ}) = 600 \text{ MJ}$$

The efficiency is the power plant is:

$$e = \frac{W}{Q_H} = \frac{(600 \text{ MJ})}{(1700 \text{ MJ})} = 0.353 \text{ } \%$$

(b) The rate of electrical energy produced, i.e., the electrical power is:

$$P = \frac{W}{t} = \frac{(600 \text{ MJ})}{(1 \text{ s})} = 600 \text{ MW}$$

**REFLECT** If the nuclear power plant is close to a city, the waste heat could instead be used to heat thousands of homes.

# Sample Problem 20-5

An inventor claims to have constructed an engine that has an efficiency of 75% when operated between the boiling and the freezing points of water. Is this possible?

$$\varepsilon \leq \varepsilon_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{(0 + 273) \text{ K}}{(100 + 273) \text{ K}} = 0.268 \approx 27\%$$

**NO !**

Air conditioner consumes  $\underbrace{1200 \text{ W}}_{\text{power}}$ ;  $\text{COP} = 3.2$ ;  $Q_c = ?$   
for  $t = 24 \text{ hours}$

72.  
Given

**ORGANIZE AND PLAN** The coefficient of performance of a refrigerator (to which we count air-conditioning systems) is the heat removed divided by the required work.

KNOWN:  $\text{COP} = 3.2$ ,  $P = 1200 \text{ W}$ ,  $t = 24 \text{ h}$ .

**SOLVE** The electrical energy (required work) consumed in 24 h is:

$$W = Pt = (1200 \text{ W})(24 \text{ h}) = 1.0 \times 10^8 \text{ J} \quad \leftarrow ; P = \frac{W}{t}$$

The amount of heat removed from the house is:

$$Q_c = \text{COP} \times W = (3.2)(1.0 \times 10^8 \text{ J}) = 3.3 \times 10^8 \text{ J} \quad ; \text{COP} = \frac{Q_c}{W}$$

**REFLECT** The higher the COP, the larger the amount of heat removed.

$W_{old}$   $\xrightarrow{e}$  \$180 ; if pump with COP = 3.1 | Bill = ?

94. **ORGANIZE AND PLAN** The coefficient of performance (COP) of a heat pump equals the heat delivered divided by the electrical energy used. This means that with a heat pump, the required electrical energy is reduced by a factor that equals the COP.

Known:  $c = \$180$ ; COP = 3.1.

**SOLVE** The monthly heating bill after installing a heat pump is:

$$\frac{c}{COP} = \frac{(\$180)}{(3.1)} = \$58$$

**REFLECT** We have assumed that the cost per kWh is constant. This is not always true: some electrical utility companies charge higher prices per kWh once a certain level has been surpassed. In such a situation, the savings would be even greater.

$$W_{old} = Q_H = \$180$$

$$COP = \frac{Q_H}{W_{new}} \Rightarrow W_{new} = \frac{Q_H}{COP} = \frac{W_{old}}{COP} = \frac{\$180}{COP}$$

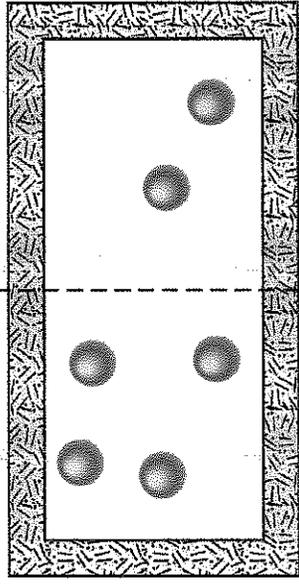
# Statistical view of Entropy

$$W = \frac{N!}{n_1! n_2!}$$

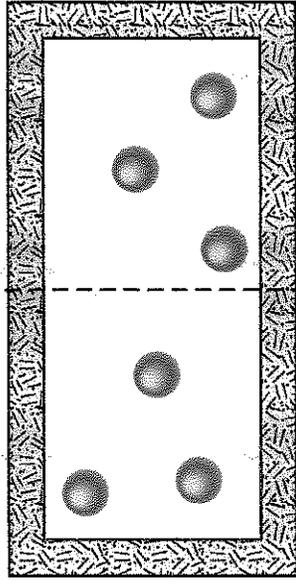
$$S = k \ln W$$

Table 21-1 Six Molecules in a Box

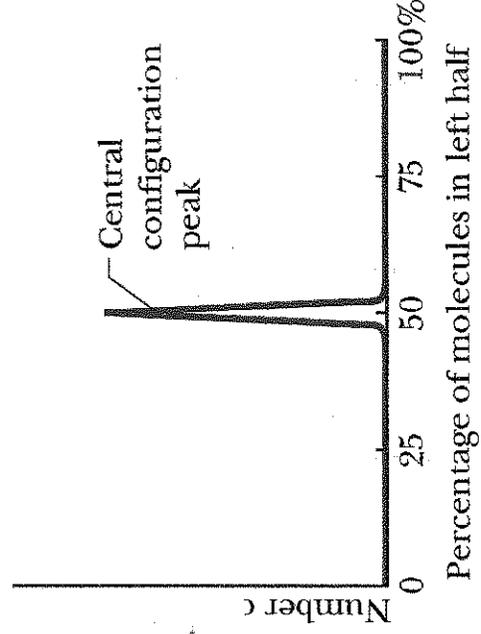
Label	Configuration $n_1$	$n_2$	Multiplicity $W$ (number of microstates)	Calculation of $W$ (Eq. 21-18)	Entropy $10^{-23}$ J/K (Eq. 21-19)
I	6	0	1	$6!/(6! 0!) = 1$	0
II	5	1	6	$6!/(5! 1!) = 6$	2.47
III	4	2	15	$6!/(4! 2!) = 15$	3.74
IV	3	3	20	$6!/(3! 3!) = 20$	4.13
Total number of microstates = 64					



(a)

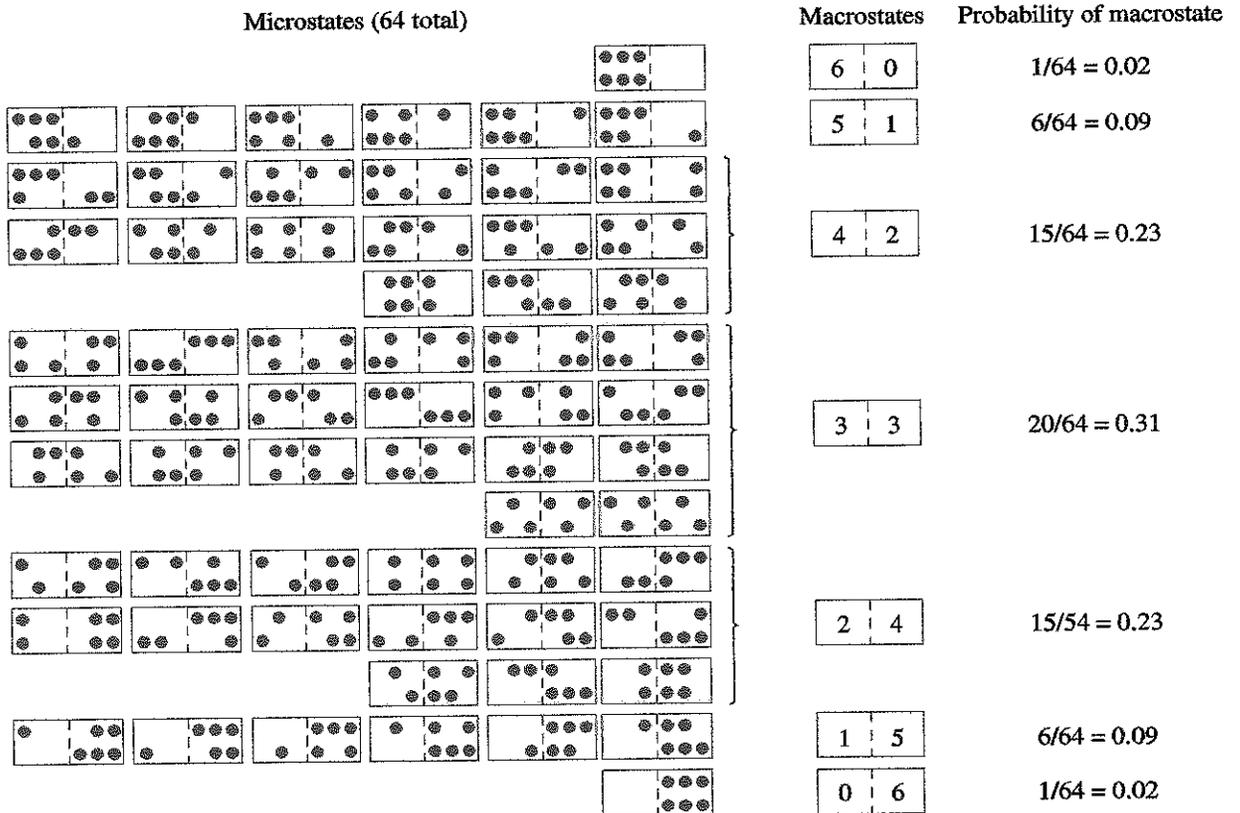
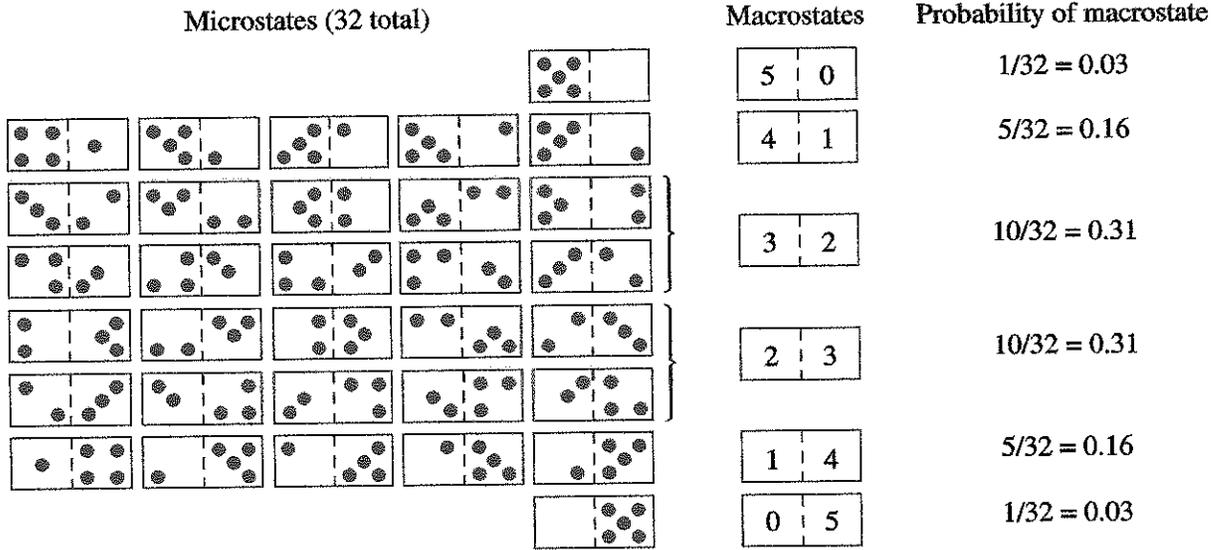


(b)



$$\ln(N!) \approx N \ln(N) - N$$

Examples of micro and macrostates for 5 and 6 molecules



$$\sum p = ?$$

77. **ORGANIZE AND PLAN** We will sum all probabilities.

*Known:* Figure in solution to Problem 76.

**SOLVE** (a) The sum of all probabilities are:

$$\frac{1}{32} + \frac{5}{32} + \frac{10}{32} + \frac{10}{32} + \frac{5}{32} + \frac{1}{32} = \frac{32}{32} = 1$$

(b) The sum of all probabilities are:

$$\frac{1}{64} + \frac{6}{64} + \frac{15}{64} + \frac{20}{64} + \frac{15}{64} + \frac{6}{64} + \frac{1}{64} = \frac{64}{64} = 1$$

**REFLECT** If you sum the ratios rather than the rounded-off decimal numbers you are guaranteed to get a sum that equals 1.

78. **ORGANIZE AND PLAN** The entropy is Boltzmann's number multiplied with the logarithm of the number of microstates in each macrostate.

*Known:* Figure in solution to Problem 76.

**SOLVE** (a) The entropy of the macrostates with 5 molecules on one side and 0 on the other side is:

$$S = k_B \ln(1) = 0$$

The entropy of the macrostates with 4 molecules on one side and 1 on the other side is:

$$S = k_B \ln(5) = 2.22 \times 10^{-23}$$

The entropy of the macrostates with 3 molecules on one side and 2 on the other side is:

$$S = k_B \ln(10) = 3.18 \times 10^{-23}$$

(b) The entropy of the macrostates with 6 molecules on one side and 0 on the other side is:

$$S = k_B \ln(1) = 0$$

The entropy of the macrostates with 5 molecules on one side and 1 on the other side is:

$$S = k_B \ln(6) = 2.47 \times 10^{-23}$$

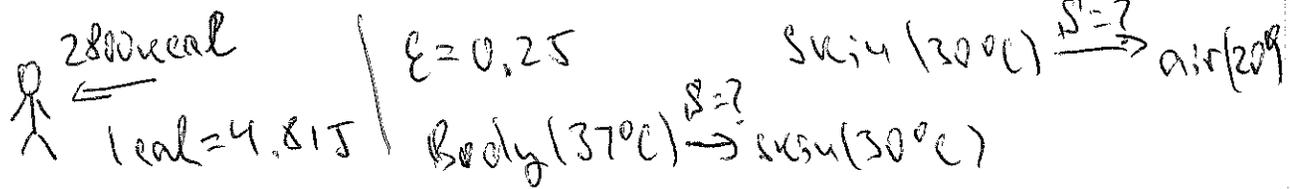
The entropy of the macrostates with 4 molecules on one side and 2 on the other side is:

$$S = k_B \ln(15) = 3.74 \times 10^{-23}$$

The entropy of the macrostate with 3 molecules on either side is:

$$S = k_B \ln(20) = 4.14 \times 10^{-23}$$

**REFLECT** The larger the number of microstates in a macrostate, the larger the entropy of the macrostate.



**98. ORGANIZE AND PLAN** The entropy change of a system is the heat flow into the system divided by the temperature. The net entropy produced is the sum of the entropy change of all systems.

*Known:*  $\Delta U = 2800 \text{ kcal}$ ;  $e = 25\%$ ;  $T_{\text{core}} = 37^\circ\text{C}$ ;  $T_{\text{skin}} = 30^\circ\text{C}$ ;  $T_{\text{air}} = 20^\circ\text{C}$ .

**SOLVE** (a) The heat flow out of the body's core is:

$$Q = (1 - e)\Delta U = (1 - 25\%)(2800 \text{ kcal}) = 2.1 \times 10^3 \text{ kcal} = 8.8 \times 10^6 \text{ J}$$

The entropy change in the body's core is:

$$\Delta S_{\text{core}} = \frac{-Q}{T_{\text{core}}} = \frac{-(8.8 \times 10^6 \text{ J})}{(37^\circ\text{C})} = -2.8 \times 10^4 \text{ J/K}$$

The entropy change in the outer skin is:

$$\Delta S_{\text{skin}} = \frac{Q}{T_{\text{skin}}} = \frac{(8.8 \times 10^6 \text{ J})}{(30^\circ\text{C})} = 2.9 \times 10^4 \text{ J/K}$$

The net entropy production in this process is:

$$\Delta S_{\text{core}} + \Delta S_{\text{skin}} = (-2.8 \times 10^4 \text{ J/K}) + (2.9 \times 10^4 \text{ J/K}) = 6.5 \times 10^2 \text{ J/K} \quad \uparrow$$

(b) When the heat is radiated away, the entropy change in the skin is:

$$\Delta S_{\text{skin}} = \frac{-Q}{T_{\text{skin}}} = \frac{(-8.8 \times 10^6 \text{ J})}{(30^\circ\text{C})} = -2.9 \times 10^4 \text{ J/K}$$

The entropy change when in the air is:

$$\Delta S_{\text{air}} = \frac{Q}{T_{\text{air}}} = \frac{(8.8 \times 10^6 \text{ J})}{(20^\circ\text{C})} = 3.0 \times 10^4 \text{ J/K}$$

The additional entropy produced when the heat is radiated away is:

$$\Delta S_{\text{skin}} + \Delta S_{\text{air}} = (-2.9 \times 10^4 \text{ J/K}) + (3.0 \times 10^4 \text{ J/K}) = 9.9 \times 10^2 \text{ J/K} \quad \uparrow$$

**REFLECT** The net entropy change of the entire process is:

$$\Delta S_{\text{core}} + \Delta S_{\text{air}} = (-2.8 \times 10^4 \text{ J/K}) + (3.0 \times 10^4 \text{ J/K}) = 1.6 \times 10^3 \text{ J/K}$$