Transformations (part 2)

Congruence Transformations (Isometries)

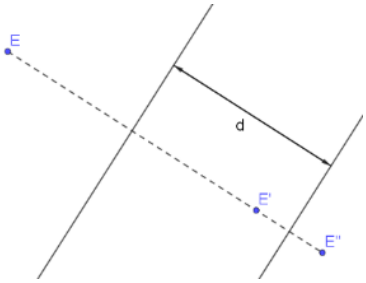
1. You should be able to list all isometries and all that is required to unambiguously determine each of them.
2. Discuss point symmetry and identity. Why do we often leave them out from the list of isometries?
3. Discuss glide reflection. Why do we want the vector to be parallel to the line of reflection?

Tools of transformations

4. You should be able to solve elementary problems (see examples at https://www.geogebra.org/m/UL59oGXJ) and provide useful instructional tips for working with typical tools such as transparency, square dot paper or technology.
5. Given a pre-image and its image, find the corresponding isometry. https://www.geogebra.org/m/jpDtBfw2 .

Composition of Isometries

6. Explain why the composition of three reflections cannot yield a translation or rotation. Try to explain it without using technology, just by referring to some properties of isometries.
7. Explain why reflection has a special position among the isometries and illustrate it on a few examples.
8. Prove that the angle of rotation generated by two reflections with intersecting lines is 2x the angle between the lines.
9. Show that the vector of translation generated by two reflections with parallel lines is twice as long as the distance between the two lines. In other words, describe the translation vector that maps E onto E'.



Solids

Classification of solids

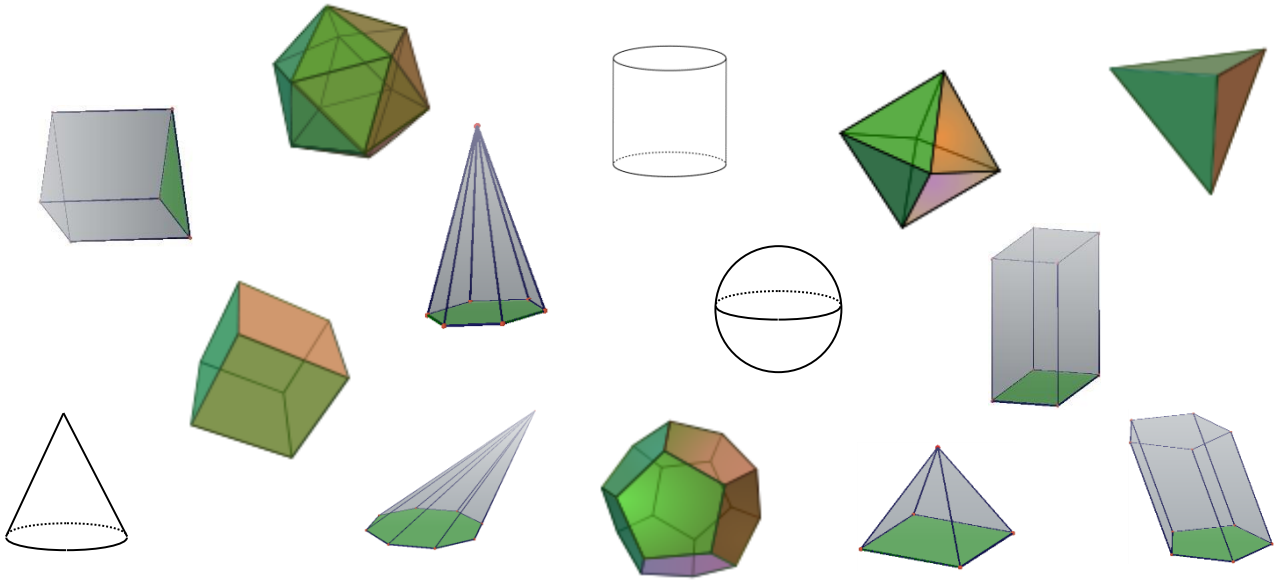
You should be familiar with the basic terminology and names: *Vertex, Edge, Face; Solid, Polyhedron, Prism (right and oblique), Antiprism, Pyramid (right and oblique), Platonic Solid, Regular Polyhedron, Archimedean Solid, Semiregular Polyhedron, Sphere, Cone, and Cylinder*.

10. Do solids have sides? Why we are not using this term when referring to solids?
11. Construct a tree diagram showing the relationships among of solids. Try to construct the classification from your memory, before you look at the diagram from your notes or on the last page.

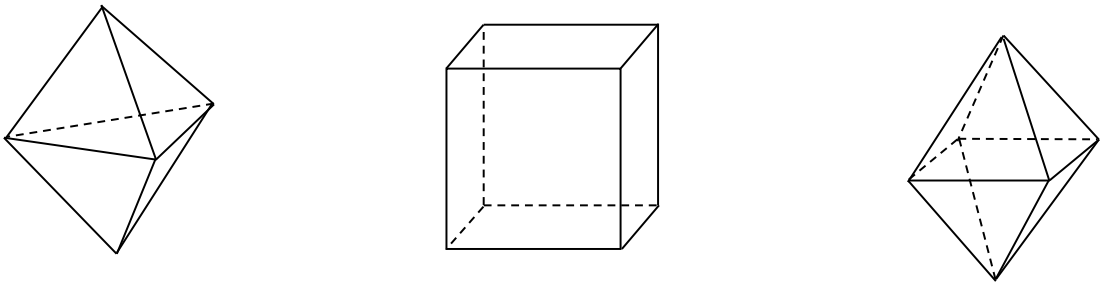
12. Compare your classification with one on the last page and fill in the descriptions or definitions of these solids and provide specific examples.
    a. What is (are) a defining feature(s) of a polyhedron?
    b. What is (are) a defining feature(s) of a prism? What kinds of prisms are there and how do they differ?
    c. What is (are) a defining feature(s) of a pyramid? What kinds of pyramids are there and how do they differ?
    d. What is (are) a defining feature(s) of Archimedean Solids and how do they differ from Platonic Solids?

Classification of solids is not a strict hierarchy and definitions may differ from literature to literature. Understanding of the basic structure of solids and knowing the defining features of the most common solids is therefore more important than "memorizing" a particular structure or definitions. You should be able to see how the structure we discussed in class can be "tweaked" to include other categories you may read about in the future. A few examples:

13. Find out what "prismatoid" and "parallelepipeds" are and explain how they relate to the solids we discussed (gray connections in the picture):
    a. Think of one reason why antiprisms should be included in prisms. Think of one reason why they should be excluded.
    b. Why are right prisms also parallelepipeds?
    c. Why are antiprisms excluded from the group of parallelepipeds?
    d. List all categories of solids to which a cube would belong.
    e. Are pyramids also prismatoids? Why or why not?
14. What would be an oblique cone and oblique cylinder?
15. Find out what deltoids are and place them in the structure.
16. Define a sphere as a set of certain points in the space.
17. Demonstrate a sphere as a solid of revolution. What 2D shape would you be rotating and where would be the axis of rotation?
18. Demonstrate a cylinder as a solid of revolution. What 2D shape would you be rotating and where would be the axis of rotation?
19. Demonstrate a cone as a solid of revolution. What 2D shape would you be rotating and where would be the axis of rotation?
20. Name the solids depicted in the following picture. Be as specific as you can (don't just state "pyramid"; specify what kind of pyramid it is). Sort out the solids using categories *polyhedron*, *prism*, *pyramid* and *platonic solid*. (Note: Some do not belong to any category and some may belong to more than one group).



21. Sketch a cube. Sketch a triangular prism. Sketch a square pyramid and tetrahedron. Sketch a cone and cylinder.
22. Name all Platonic Solids and describe them (give number of faces and their shape).
23. Are the depicted solids regular, semiregular, or none of the above? Explain. (All faces in the depicted solids are regular polygons: congruent equilateral triangles and squares)



Volume and Surface Area

24. What is a net of a solid? Draw a few nets of a cube. Draw a net of a rectangular prism. Draw a net of a square pyramid. Draw 2 visually different nets of a cone. (You will be allowed to use paper and scissors or Polydron to verify that your pattern is actually a net of that solid.)
25. Describe solids given their nets https://www.geogebra.org/m/pstiFBiv
26. You should be familiar with basic formulas for the volume and surface area. The focus is on understanding how these formulas are developed (not on memorizing them).
    a. Explain why the volume of a prism can be calculated as the area of the base x height. (you may draw block buildings, cubes, etc.)
    b. Based on the above, explain why the volume of a rectangular prism can be calculated as length x width x height

c. Use the above to reason about the volume of a cylinder.

d. How is the volume of a pyramid related to the volume of a prism with the same base and height? How can students discover it?

e. Use the above to reason about the volume of a cone.

f. Show how the formula for the surface area of the cylinder can be derived.

g. Generalize the above to give the formula for the surface area of a prism.

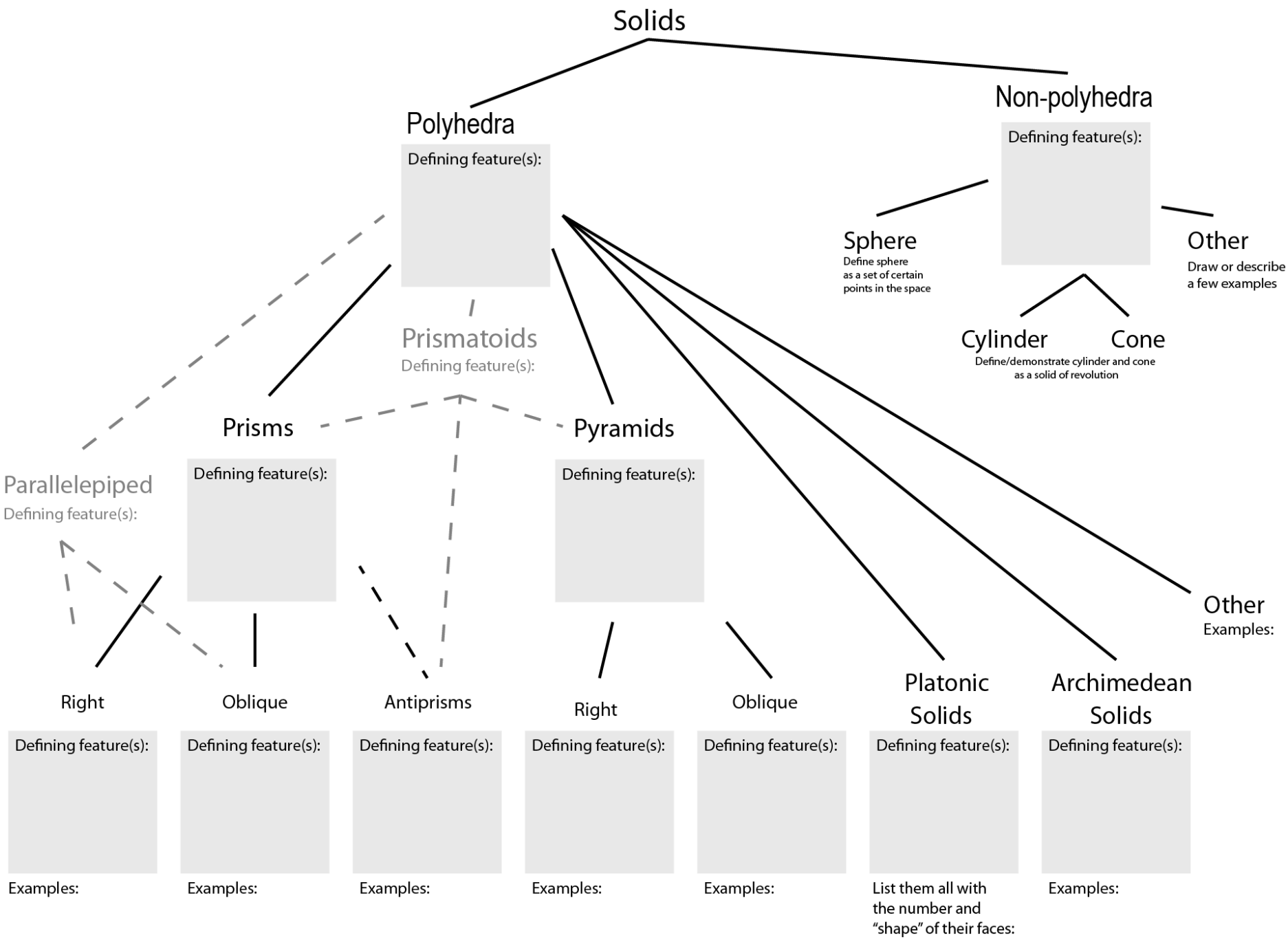
h. Explain (derive) the formula for the surface area of a cone.

27. Derive the formula for the volume of the sphere appropriate for a secondary class (no calculus). Explain what your students already know, what you would ask them to do and how do you expect them to derive the formula.

28. There is interesting relationship between the surface area of a cylinder and its inscribed sphere. How can you use it to derive the surface area formula for a sphere?

29. Explain the "peeling an orange" activity to derive or estimate the surface area of a sphere. Discuss the positives of the activity as well as possible pitfalls that you need to be ready for.

30. You should be able to find volumes and surface areas of the most common solids and their combinations. Examples are in Geogebra Book*:* https://www.geogebra.org/m/cr7NBxgw (See the chapter 3.Measurement in the space, problems #3-8. Answers are given for you to check your work).


Solids
Polyhedra
Defining feature(s):
Non-polyhedra
Defining feature(s):
Sphere
Define sphere as a set of certain points in the space
Other
Draw or describe a few examples
Cylinder
Cone
Define/demonstrate cylinder and cone as a solid of revolution
Prismatoids
Defining feature(s):
Prisms
Defining feature(s):
Pyramids
Defining feature(s):
Parallelepiped
Defining feature(s):
Other
Examples:
Right
Oblique
Antiprisms
Right
Oblique
Platonic Solids
Archimedean Solids
Defining feature(s):
Defining feature(s):
Defining feature(s):
Defining feature(s):
Defining feature(s):
Defining feature(s):
Defining feature(s):
Examples:
Examples:
Examples:
Examples:
Examples:
List them all with the number and "shape" of their faces:
Examples: