MTH 362: Study Guide for the Exam 2

The exam will take place in the computer lab. You will have a choice to work on your TI-Nspire or on a computer and submit (1) a test sheet with your answers (handwriting is fine) and (2) .tns or .ggb files. You will submit the file by e-mail (wait for a confirmation that I received your file).

Constructions of polygons and quadrilaterals

You should be able to construct polygons (with made-up names) if given their minimal definitions and answer questions regarding these shapes.

1. "House": A pentagon with two adjacent right angles.
    a. Q1: Can a "house" have two diagonals that are perpendicular to each other?
    b. Q2: Can a "house" be a concave polygon?
    c. Q3: Is a regular pentagon a house?

2. "Funhex": A hexagon with all pairs of opposite sides congruent.
    a. Q1: Is a regular hexagon a Funhex?
    b. Q2: Can Funhex be a concave polygon?

Constructions of quadrilaterals

You should be able to construct quadrilaterals (kite, trapezoid, isosceles trapezoid, parallelogram, rhombus, rectangle, square) given their minimal definitions (you do not have to memorize the definitions, they will be provided). Make sure your object is not over- or under- constrained.

You should be able to see how minimal definitions govern the construction of quadrilaterals and construct some quadrilaterals from their minimal definitions:

1. Construct a kite as
    a. A quadrilateral with two distinct pairs of congruent adjacent sides.
    b. A quadrilateral that is symmetrical with respect to one of its diagonals.
    c. A quadrilateral with two congruent adjacent sides and one diagonal being an angle bisector of the angle formed by these two congruent sides.
2. Construct a rhombus as
    a. A quadrilateral with four congruent sides.
    b. A parallelogram with two congruent adjacent sides.
3. Construct a rectangle as
    a. A quadrilateral with 3 right angles.
    b. A parallelogram with one right angle.

You should be able to use your constructions and triangle congruence to answer questions about the quadrilateral's properties. Keep in mind that those properties must hold for all quadrilaterals included in the category, not just the one displayed on your screen. If a property does not hold for the given quadrilateral, or if it holds only sometimes, make sure you provide examples and counterexamples. If it holds, briefly justify your answer.The following charts includes hints for justifying the properties: http://people.cst.cmich.edu/marci1t/geometry_ebook/04_polygons%20and%20quadrilaterals/properties/quadrilaterals-properties-chart.htm .

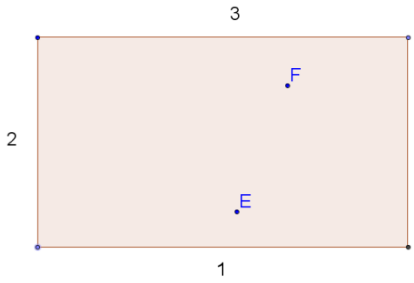
1. Kite: a quadrilateral with two distinct pairs of adjacent congruent sides.
    a. Explain the meaning and importance of the word “distinct” in the definition.
    b. Are squares kites by this definition?
    c. Q1: Are the diagonals always perpendicular?
    d. Q2: What other observation about kite’s diagonals can you make?
    e. Justify the formula for the area of a kite (diagonals are known).
2. Trapezoid: a quadrilateral with at least one pair of parallel sides.
    a. We talked about two different definitions of a trapezoid (we referred to them as “modern”or inclusive and “traditional” or exclusive). Discuss the differences and explain why the “modern” definition is becoming a preferred way to define a trapezoid.
    b. We talked about two different definitions of a trapezoid (we referred to them as “modern”or inclusive and “traditional” or exclusive). Discuss the differences and explain why the inclusive definition is becoming a preferred way to define a trapezoid. Is the inclusive definition appropriate for students of all ages? Explain.
    c. What is a trapezium? If you are not sure, find its definition on the Internet.
        i. Look up the definition of a trapezoid and trapezium in British English. What did you find?
    d. Q1: Certain pairs of adjacent angles have a special property. What is the property and which angles does it apply to? Label the angles to be able to refer to them in your answer.
    e. Q2: Diagonals split a trapezoid into 4 triangles. Among those 4 triangles, there is one pair of similar triangles. Identify the two and explain why they are similar. The other two triangles have the same area. Explain why.
    Label the triangles to be able to refer to them in your answer.
    f. Justify the formula for the area of a trapezoid in two different ways, using (1) the triangle area formula; and (2) the parallelogram area formula.
3. Isosceles trapezoid: a trapezoid with one pair of congruent base angles.
    a. Prove that diagonals are congruent. You may assume that the legs of isosceles trapezoid are congruent and angles in the definition are congruent.
    b. Prove that among 4 non-overlapping triangles created by the diagonals, there is one pair of similar and one pair of congruent triangles. You may assume that diagonals are congruent, the legs of isosceles trapezoid are congruent and angles in the definition are congruent.
4. Parallelogram: a quadrilateral with two pairs of parallel sides
    a. Show that the opposite sides of a parallelogram are congruent.
    b. Q1: Are the diagonals congruent?
    c. Q2: Are the diagonals perpendicular to each other?
    d. Q3: Are the diagonals bisecting each other?
    e. Q4: Is there another property of diagonals worth mentioning?
    f. Justify the formula for the area of a parallelogram.
5. Rhombus: a quadrilateral with four congruent sides.
    a. Show that rhombus is a parallelogram.
    b. Q1: Are the diagonals congruent?

c. Q2: Are the diagonals perpendicular to each other?
d. Q3: Are the diagonals bisecting each other?
e. Q4: Diagonals split the rhombus into 4 triangles. They all share certain properties. Name at least one such property.

6. Rectangle: a quadrilateral with three right angles.
   a. Show that a parallelogram with one right angle is also a rectangle.
   b. Q: Specify two properties of the diagonals.
7. Square: a quadrilateral with four congruent sides and one right angle.
   a. Show that all angles in the square are right angles.
   b. Q: What might be an alternate formula for the area of a square that does not involve the side(s) of the square? (*See if square is a kite.*)

Isometries

Review the use of isometries to solve geometry problems.



- "Thirsty horse problem"
- Billiard problems: Find the shortest path from the cue ball (E) to the target ball (F) so that it bounces from three edges in the order indicated in the picture. Explain why your path is the shortest possible.
- Explain why reflection has a special position among other isometries and illustrate it on a few examples.
- Look at the following arrangement of lines of reflection a, b, c. If you compose these line reflections in this order (first a, then b, then c), what is the resulting isometry? Can it be a translation or rotation? Why? (If you want to play with the composition, draw the lines in GeoGebra/TI-nspire, then draw a triangle and find the resulting image. Look at your original triangle and its final image and answer the questions).