MTH 362: Study Guide – Constructions and Triangles

The exam will take place in the computer lab. You will have a choice to work on your TI-Nspire or on a computer and submit (1) a test sheet with your answers (handwriting is fine) and (2) .tns or .ggb files. You will submit the file by e-mail (wait for a confirmation that I received your file).

THE SYSTEM OF EUCLIDEAN GEOMETRY

1. Describe the "building blocks" of Euclidean geometry. Who was the first to propose geometry as a system and when was it? (approximately)
2. Look at the high school CCSS and identify theorems, the proof of which should be discussed in high schools.
3. Prove the Vertical Angles Theorem. Prove the Interior Angle Sum in a Triangle Theorem.

CONSTRUCTIONS

Recall that many construction "tools" available in GeoGebra or TI-Nspire (midpoint, perpendicular line, etc.) are shortcuts and are not allowed in strictly elementary Euclidean constructions.

4. Discuss what we mean by elementary Euclidean constructions and how they are different from constructions that are possible by using technology.
5. Name two tools of elementary Euclidean constructions. Which Euclid's postulates are referring to these tools?
6. Review the elementary constructions we talked about (Midpoint, Angle bisector, Perpendicular line (2 situations), Transferring an angle, Parallel line) making sure you are only using a straightedge and collapsible compass or compass. Record your construction steps so that someone can follow it. You may use Applets from my website to test the correctness of your constructions: https://www.geogebra.org/m/GUDwUySb
7. CCSS for High School Geometry list several different tools and methods to make formal geometric constructions (Compass&Ruler, Paper Folding, Technology). Pick a construction and demonstrate it using 3 different methods. Briefly discuss pros/cons or similarities/differences of these methods.
8. Can elementary constructions mentioned in #6 be done by paper folding? Are there some that cannot be done by paper folding? Be specific and explain your reasoning.

TRIANGLE CONSTRUCTIONS

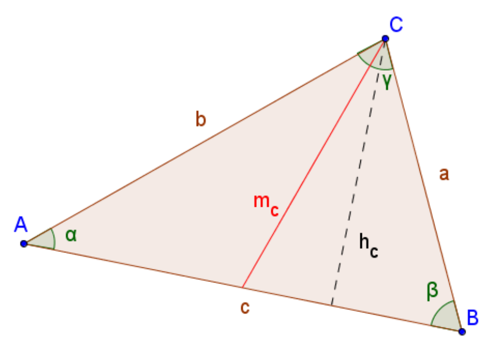
Notation: (You may use different labels as long as your picture is readable and unambiguous.)

Sides: lower case letter corresponding to the opposite vertex

Medians: m with the side to which it is drawn in the subscript

Altitudes: h with the side to which it is drawn in the subscript

Angles: Greek lower case letters (α, β, γ, …)



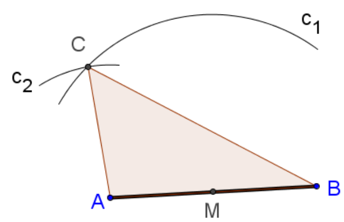
For construction problems, you should include the following:

- Construct the figure. Make sure it is a valid construction, not just a drawing.
- Play with your construction and discuss the number of solutions. You should have as complete discussion as possible. For example, in class we discussed the construction of a triangle, if given the side AB and AC and height to AB. Possible discussion may look like this:

  0 solutions: when the altitude is longer than the side (or simply $h_c > b$)
  2 solutions (two isosceles triangles): $h_c = b$
  4 solutions: $h_c < b$

9. Construct triangles given the following. (The first object is given in the center of the screen, other two are on a side):
   a. a, b, c
   b. a, b, m_b
   c. a, b, h_a
   d. a, m_a, h_a
   e. Comment on the student's solution depicted on the right.

AB is given. Construct a triangle ABC so that |AC| = 3cm and the height falling on the side AB is 4 cm long.



Construction Protocol:
1. M | M is the midpoint of AB
2. c_1 | c_1(M, 4cm)
3. c_2 | c_2(A, 3cm)
4. C| {C} = $c_1 \cap c_2$
5. ΔABC

Triangle Constructions Applets:
https://www.geogebra.org/m/CccA5Zhj

OTHER TRIANGLE FACTS AND THEOREMS

10. A student says: Any median of a triangle is always longer than the height falling on the same side. Comment on the student's claim.
11. Comment on a student's claim: An acute triangle is a triangle with an acute angle.

Triangle Inequality

12. Explain the difference between triangle inequality theorem and criterion.
13. Suggest two activities (one with manipulatives, one with technology) that will help students discover the theorem or criterion and explain why it works.

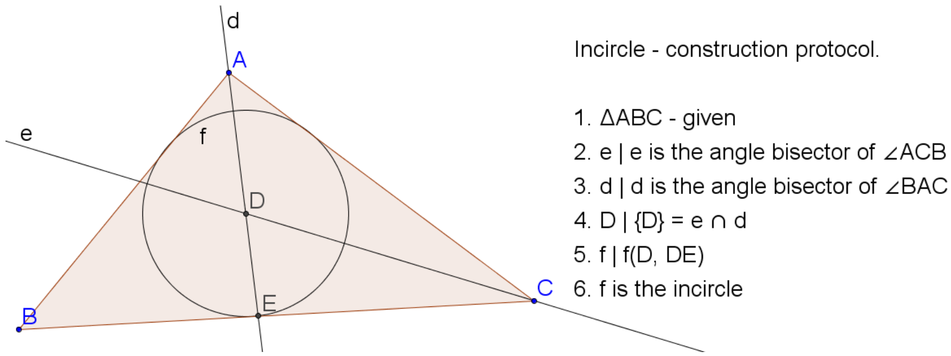
Centers and Circles in a Triangle

You should be able to identify or construct the following objects in a triangle:

- Altitudes, Heights and Medians
- Orthocenter, Circumcenter, Incenter, Centroid
- Circumcircle, Incircle

(You may use Applets to test your constructions)

14. Comment on the student's solution below. If the students solved the problem using technology, what would you suggest to do to check the validity of the construction?



15. Draw a circle, hide its center. Use common construction tools (not limited to compass and ruler) to find the center of your circle. Are you drawing on any knowledge about triangles? (You are allowed to use any tool on TI-Nspire. In GeoGebra, you are not allowed to use the "Midpoint or Center" tool for finding the centers of the circle. You can only use it to find midpoints of line segments if needed.)

Pythagorean theorem

16. Formulate the theorem and prove it in at least 3 different ways.
17. Discuss two ways to generalize the Pythagorean theorem.

Midsegment theorem

18. Formulate the theorem and prove it.

Applet with hints: https://www.geogebra.org/m/TeXKJjVc

Midsegment Quadrilateral

19. Formulate observations related the following properties of midsegment quadrilateral and **justify your observation.** Play with Geogebra as needed.
    i. Midsegment quadrilateral is always a ………….. *(insert a type of quadrilateral)*.
    ii. Midsegment quadrilateral will be a rectangle if the original quadrilateral ………… *(name such quadrilateral or specify its property)*.
    iii. Midsegment quadrilateral will be a square if the original quadrilateral ……….. (*name such quadrilateral or specify its property*).
    iv. How does the area of a midsegment quadrilateral relate to the area of the original quadrilateral?
    v. What can you say about the perimeter of the midsegment quadrilateral? Does it relate to something in the original one?

Properties of medians

20. Describe the position of the centroid on a median. Justify your claim.
21. Explain why all medians are concurrent.
22. Show that a median splits a triangle into two triangles of equal area.
23. If a median of a triangle is drawn, prove that the segments drawn from the other vertices perpendicularly to this median have the same length.