Toward A Socially Optimal Wireless Spectrum Management

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***Abstract*—It is widely recognized that the wireless spectrum is a scarce and limited resource and that the present practice of static spectrum allocation and exclusive licensing is inefficient. While many models have been proposed, approaches generally either focus on profit maximization of individuals (such as the government or users) or maximization of spectrum utilization. In this paper, we consider an efficient, or socially optimal, spectrum sharing that consists of three objectives: full (quantitative) utilization, effective (qualitative) utilization, and zero interference. Through a comparative study of these models using suggested objective criteria, we show a hybrid model consisting of a dynamic spectrum market and dynamic spectrum access supported by cognitive radio technologies that can achieve the social optimum. The dynamic spectrum market enabled by a benevolent social arbitrator has fundamental differences from existing dynamic market models in that primary licensed user is not involved in the process of allocating underused spectrum. Moreover, the motivation of social arbitrator is to reach socially optimal allocation of spectrum resources rather than to maximize profit or revenue of individuals.**

I. Introduction

Radio spectrum like other resources is limited and scarce in the provision of wireless telecommunications services. Seeking a better utilization of these spectrum resources has become a pressing issue [1]. Historically, the allocation of spectrum resources has been strictly regulated because of externalities in spectrum usage such as electromagnetic interference. Traditionally, the practice has been centralized, static, and wholesale type of spectrum allocation such as the spectrum auction, in which governments assign exclusive rights to transmit signals over specific spectrum, and this practice is considered inefficient as it leads to under-utilization of spectrum resources [1], [2].

The under-utilization of spectrum has stimulated the engineering, economics, and regulatory communities in searching for better spectrum management policies and techniques. Three major models have been developed to complement or to replace the current auction model of the Federal Communications Commission (FCC): the dynamic spectrum market model [3]–[7], the cognitive radio (CR) model [8], [9], and the spectrum commons model [10], [11]. The dynamic spectrum market model requires the government assigning property rights to license holders who can resell unused spectrum while the cognitive radio model allows licensed-exempt use by secondary users of frequency owned by a licensed primary user. The spectrum commons model employs open sharing among peer users with an equal right of access as the basis for managing a spectral region. However, it is critical to note these models focus either on the full utilization of spectrum or the profit maximization of governments or primary users, but not the *efficient* (or *socially optimal*[1]) use of spectrum resources from an economic perspective. The dynamic spectrum market model will improve spectrum utilization but is hard to reach full utilization of spectrum due to the deviation of private incentives from social incentives when primary users actively participate in dynamic spectrum markets [12]. The cognitive radio model may reach full utilization but cannot guarantee the *effective* use of spectrum if secondary users with heterogenous valuation of spectrum usage have equal access to licensed spectrum.

Our approach focuses on a socially optimal spectrum resource management system with three important objectives: full (quantitative) utilization, effective (qualitative) utilization, and zero interference. Through comparing the advantages and disadvantages of the four aforementioned models (the FCC model, the dynamic spectrum market model, the cognitive radio model, and the spectrum commons model), we propose a hybrid model combining the dynamic spectrum market and the dynamic spectrum access supported by the cognitive radio technology that outperforms the existing models in reaching a good balance of the three objectives of efficiency. The hybrid model allows anyone to have access to unused licensed spectrum resources with a potentially positive access cost (depending on changing congestion conditions) payable to a social arbitrator (e.g. the government regulator such as the FCC). Primary users are excluded from the process of reallocation of unused spectrum to maximize the possible supply of residual bandwidth. It is important to note that the goal of the benevolent social arbitrator is to *maximize social welfare of spectrum usage*, not to maximize the profits of any party. The optimal equilibrium cost provided in real time by the social arbitrator is the *minimum* cutoff price that induces only those users with higher valuation of spectrum usage to

[1] An allocation of scarce resources is considered *efficient* if the *social welfare* of using the resources is maximized. In this paper, "efficiency" and "social optimum" are used interchangeably.

actually use the resources. In the case of sufficient supply of spectrum resources, the price can be set at zero, essentially free open access.

The contribution of the paper is both pointing the direction of a socially optimal utilization of wireless spectrum from an economics perspective and defining three objective criteria to reach the social optimum. We illustrate of how a hybrid model of dynamic spectrum market and dynamic spectrum access enabled by cognitive radio technologies can actually achieve the social optimum by taking good balance of full and effective utilization of the limited rescouses. The rest of the paper is organized as follows. Section II reviews and compares the key features of four major models of spectrum resource management: the current license auctioning model of the FCC, the dynamic spectrum market model, the cognitive radio model, and the spectrum commons model, using the three objectives as criteria of judgment. Section III proposes the hybrid model in details and derives the optimal access cost that may lead to the efficient, or socially optimal, result of spectrum allocation. The differences between existing and the proposed dynamic market component are contrasted. A scenario of how to find the optimal price through a case study is also discussed. Finally, Section IV concludes the paper.

II. A Welfare Evaluation of Major Models of Spectrum Management

In this section, we use the three objectives of efficient spectrum allocation to compare the four major models of spectrum resource management: the FCC model, the dynamic market model, the cognitive radio model, and the commons model. The three comparison criteria used in the following analysis are:

- *Full (quantitative) utilization*: Utilization maximization, i.e. the demand for spectrum resources is satisfied to the maximum.
- *Effective (qualitative) utilization*: The spectrum resources are only allocated to those users who valuate and benefit the most from the spectrum usage.
- *Zero interference*: No overuse so that users do not interfere with each other.

A. License Auctioning: the FCC Model

In the United States, the FCC has been using spectrum licenses to allot spectrums to applicants. A licensed regime provides the certainty needed to ensure broad investment in the band as can be provided by exclusive licensed use. It is recognized that compared to approaches such as comparative hearings and lotteries, market-based mechanisms such as auctions are more efficient for spectrum allocation [1]. In a well-designed auction, everyone has an equal opportunity to win and the spectrum is sold to bidders who value it the most, hence likely to use it most effectively. However, full utilization will be satisfied only if the bandwidth demand by the primary user is greater than or equal to the bandwidth supply. As it has been widely shown, licensed spectrums are often unused or under-utilized, resulting in significant waste of spectrum resources [1], [2].

B. The Dynamic Spectrum Market Model

The under-utilization of spectrum has stimulated a large bed of literature exploring the issue of dynamic spectrum sharing and management [3]–[7], [13]–[15]. The dynamic spectrum market model we refer to is a combination of spectrum property rights (with exclusive-use) and hierarchical spectrum markets, i.e., the spectrum bands license holders have the rights to resell part of their unused spectrum to secondary users for profit. A hierarchical access structure can be established to coordinate primary and secondary users, thus limiting the interference perceived by primary users. One advantage is such sharing is not mandated by the regulation policy, and economy and market will play an important role in driving toward the most profitable (and hence effective) use of spectrum resources.

While well designed, dynamic spectrum markets will create incentives for license owners to share spectrum, such markets are unlikely to eliminate under-utilization from the root because transaction costs of spectrum buyers (secondary users) and sellers (primary users) can be significant and private incentives of license holders may deviate from social incentives [12]. To fully utilize spectrum, flexible short-term secondary licenses are needed on infinitely small slots in terms of the amount of spectrum, the time windows and the area coverage. It is cumbersome for license holders to fully identify the reusability of the spectrum in a very fine granularity. The delay in negotiating and finalizing contracts in auction market can also be problematic when both buyers and sellers are self-interested. A dynamic spectrum market will only arise if the transaction cost of license holders is less than the value of the spectrum to secondary users net of the transaction cost of secondary users. Study [12] also shows the transition to a property rights model for spectrum is far more complex than commonly portrayed, and secondary market for spectrums can hardly capture and fully accommodate the temporal and spatial variations in the radio environment in a timely manner.

C. The Cognitive Radio Model

Cognitive radios and opportunistic spectrum access [8], [9] seek technical solutions to the under-utilization problem and do not necessarily lead to any definite design of regimes. While cognitive radio users are capable of accessing both the licensed and the unlicensed spectrums [16], the cognitive radio model we refer to is a licensed system plus non-interfering open access by unlicensed users, which is in line with the common understanding of what cognitive radio techniques shall enable. The government assigns license holders the guaranteed priority in using licensed spectrum while secondary unlicensed users have an equal access to the unused or underused licensed spectrum, given no interference. Cognitive radios create increased efficiency by dynamically allocating spectrum. It differs from the dynamic spectrum market model in that the access is open to any non-interfering usage rather than a limited number

TABLE I
COMPARISON OF SPECTRUM RESOURCE MANAGEMENT MODELS

Model \ Objective	Full (*Quantitative*) Utilization	Effective (*Qualitative*) Utilization	Zero Interference
FCC Model	more inferior	primary user only	Yes
Dynamic Spectrum Market Model	inferior	primary user + ranked secondary users	Yes
Cognitive Radio Model	Yes	primary user + unranked secondary users	Yes
Spectrum Commons Model	Yes	unranked users	Yes (if used with CR)

of secondary users to which license holders sell for profit, thus can perform better in fully utilizing available spectrum resources.

It is interesting to observe, however, such a regime cannot guarantee the most *effective* use of spectrum resources since everyone has an equal access to the unused spectrum. If cognitive radio technology is neutral, secondary users may be randomly selected. In cases when users with less valuation for spectrum usage were selected, efficiency would not be achieved as the resources were not used in the most productive way.

D. The Spectrum Commons Model

The spectrum commons model gives users license-exempt access to spectrum, which is open to all and free from either government or private control [10]. The commons model challenges the exclusive use of spectrum by claiming that new spectrum sharing (cognitive radio) technologies allow a virtually unlimited number of persons to use the same spectrum without causing each other interference. The commons model is not an alternative to command-and-control regulation, but in fact shares many of the same inefficiencies of that system as a commons must be controlled either by private actors or by the government [11]. In addition to the resource overusage problem characterized by the "tragedy of the commons", this extreme commons model can be inefficient by itself. The commons model cannot guarantee the effective use of limited spectrum resources: when all potential users of the same spectrum have an equal access, the spectrum may be actually used by users who value the spectrum less.

E. The Comparison of the Four Models

As discussed above, none of the four models is optimal characterized by the three primary objectives. The FCC's auctioning of exclusive licenses will avoid interference, assure a high quality of service, and foster investment in the band, but not every channel in every band is fully utilized. Market-based dynamic access has the potential to increase spectrum utilization, but it can be costly and may suffer from misaligned incentives. Cognitive radio technology enables licensed-exempt use of frequency owned by a licensed party but it cannot guarantee the most effective use of spectrum resources. Managing spectrum as a commons can satisfy the full utilization and zero interference objectives (if used with cognitive radio technology) but it cannot guarantee the effective utilization of the spectrum. Regarding effective utilization, the spectrum commons model is inferior to the cognitive radio model as the latter can at least guarantee the prioritized use of the spectrum by the license holder, who values the spectrum the highest.

Table I summarizes the above four models regarding satisfying the three objectives of efficient allocation of spectrum resources. It is not difficult to see the tradeoff is between full utilization and effective utilization. The dynamic spectrum market model is superior to the FCC model regarding full utilization, but it is less competitive to the cognitive radio model on this regard; the cognitive radio model is superior to the spectrum commons model regarding effective utilization, but it is less competitive than the dynamic spectrum market model on this regard. Intuitively, an improved model can be a hybrid of the dynamic spectrum market model (for effective utilization) and the cognitive radio model (for full utilization).

III. MODELING ANALYSIS: SOCIAL OPTIMUM OF SPECTRUM ALLOCATION

While many interests have arisen in dynamic spectrum sharing with market forces, the objectives primarily are either to maximize the profit or revenue of license holders [4], to maximize the primary user's utility [7], to maximize the profit of all secondary users [6], or to maximize auctioneer's revenue [5]. None of the objectives is necessarily consistent with social optimum. Instead, we formulate a framework in which a social arbitrator (e.g., government regulators such as the FCC), rather than the primary user, coordinates secondary users for accessing the residual licensed spectrum of the primary user (Figure 1). There are two major differences with this dynamic spectrum market component compared with other dynamic market models. First, since the primary user is guaranteed the prioritized power to get access to the licensed spectrum, the primary user is neutral to the social arbitrator's choice and therefore not involved in the process of allocating unused or underused spectrum. Second, rather than to maximize profit or revenue of individuals, the motivation is totally different. The social arbitrator is *benevolent* whose motivation is to reach socially optimal allocation of spectrum resources.

On the other hand, the dynamic spectrum access component allows open-access enabled by cognitive radio technologies. It is important to observe, however, that in order to select the most *effective* secondary users, a *cost* of access has to be implemented so market forces can work to reveal secondary users' private valuation of spectrum usage. One implementation of the proposed hybrid regime can be a license

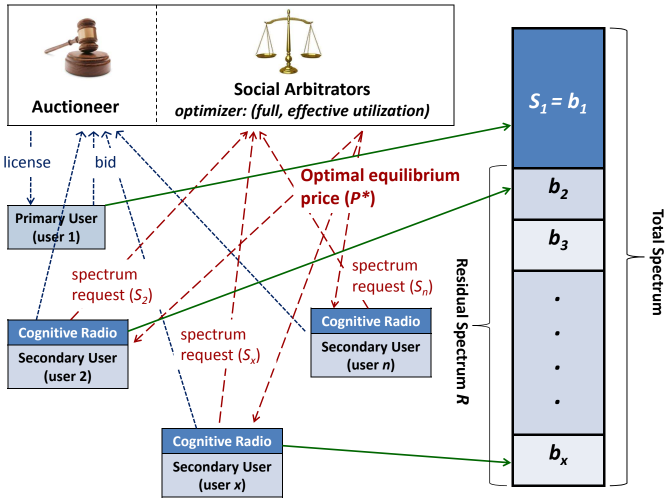

Fig. 1. Architecture for an efficient dynamic spectrum sharing. The primary user is interest-neutral and is not involved in the process of allocating unused spectrum. Cognitive radios allow dynamic and non-interference access among secondary users while a real-time trading market determines an optimal equilibrium cost for full and effective use of residual spectrum.

auctioning system plus open access with a varying price based on instantaneous changes in demand and supply of residual spectrum. Such a regime would become the cognitive radio model if the price is set to zero when the supply of residual spectrum is sufficient to satisfy all secondary users' requests for spectrum. Thus the objective of the model is not to maximize revenue from collecting fee, rather, the fee should be the *minimum* cutoff price that is sufficient to select all secondary users with higher valuations to maximize spectrum utilization.

Let s_i denoting the requested spectrum size by user i be non-negative ($s_i \geq 0$) and $S = \sum_{i=2}^{n} s_i$ be the total demand for the residual spectrum by secondary users. The supply of residual spectrum, $R = \overline{S} - s_1$, varies over time depending on the actual usage of the spectrum by the primary user. Social arbitrators collect information from secondary users regarding their demand for the spectrum size and price they are willing to pay. Based on the collected information, the social arbitrator provides an access cost according to unit-price function $p(S, R)$ and sends the price feedback to secondary users, who access the spectrum of their requested size at the given price. The pricing function of the social arbitrator to charge secondary users is assumed to be non-negative and non-decreasing for $S > 0$:

$$p(S, R) = p(\sum_{i=2}^{n} s_i, R) \tag{1}$$

where $\partial p/\partial S \geq 0$ and $\partial p/\partial R \leq 0$. The secondary users are not price-discriminated, i.e., they are all charged with the same unit rate (minimum cut-off price). With such a non-constant pricing, the spectrum cost for each secondary user not only depends on his/her own spectrum size request but also depends on other secondary users' requests and the residual spectrum left after serving the primary user.

The valuation of the spectrum by a secondary user i depends on the profit (i.e., revenue minus cost) of user i for using the spectrum, denoted as π_i. The revenue of secondary user i is $r_i \cdot s_i$ where r_i is user i's per-unit-spectrum revenue which is positively related to the user's spectrum usage efficiency. The cost of spectrum allocation for user i is $p(S, R) \cdot s_i$. The profit of user i is therefore

$$\pi_i = (r_i - p(S, R)) \cdot s_i \tag{2}$$

where $(r_i - p(S, R))$ is the per-unit-spectrum profit for user i. Since the social arbitrator charges an equal price to all secondary users, the ranking of secondary users' valuation of the spectrum resources is identical to the ranking of secondary users' revenue generated from each unit of the spectrum usage, which depends on the users' spectrum usage efficiency. If those secondary users with higher valuation of the spectrum are the ones whose demand for the residual spectrum is satisfied, the effective usage of the spectrum would be achieved.

The demand for spectrum by secondary users vary with secondary users' needs. The supply of the residual spectrum is also varying according to the primary user's instantaneous use of the spectrum. As secondary users have the option of not accessing the spectrum thus avoiding paying the cost if the feedback price exceeds the revenue, i.e., $p(S, R) > r$, they are truthful when requesting for spectrum to the social arbitrator.

The social arbitrator's optimization problem is formulated in the following objective function (Equation 3) to set a price $p(S, R)$ based on the spectrum request information collected from secondary users to reach full and effective utilization of the residual spectrum:

$$minimize : |R - \sum_{i=2}^{n} b_i| \tag{3}$$

where b_i is the actual spectrum usage by user i, and

$$b_i = \begin{cases} s_i, & \text{if } p(S, R) \leq r_i, \\ 0, & \text{if } p(S, R) > r_i. \end{cases} \tag{4}$$

If secondary users are ranked as $r_2 \geq r_3 \geq ... \geq r_x \geq ... r_{n-1} \geq r_n$, the optimal price would be $p^*(S, R) = r_x$ such that $\sum_{i=2}^{x} b_i = R$. $p^*(S, R)$ is by nature a cutoff price, the minimum price that prevents the secondary users with lower valuation of the spectrum from using the spectrum. To avoid interference, the demand by the cutoff user x may only be partially satisfied.[2] In the case of $S \leq R$, $p^*(S, R) = 0$, and all secondary users will have free open access to the unused licensed spectrum.

A. Economic Justification and Evaluation of Social Optimum of Spectrum Allocation

The social optimum of wireless spectrum utilization can be illustrated in Figure 2. The left vertical axis is the private

[2]To avoid partial service, the price can be easily adjusted to equal $r_{(x-1)}$.

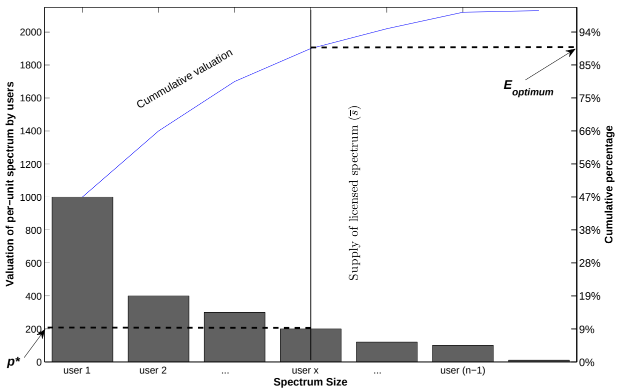

Fig. 2. A Pareto chart showing efficient (socially optimal) spectrum allocation. The widths and heights of descending bars indicate the amount and the valuation of the spectrum bandwidth demand by heterogeneous users, respectively. $E_{optimum}$ is the highest level of effectiveness (cumulative social valuation) that can be reached at the optimal price p^*.

valuation of per-unit spectrum by potential users. The n users are ranked by their valuations of per-unit spectrum usage from the highest to the lowest, with user n has the lowest valuation of the spectrum usage. When potential users are ranked according to their individual valuations of some spectrum, we can derive the private value curve (analogous to the demand curve in the market supply-and-demand analysis) of the spectrum, as the descending bars illustrated in Figure 2. The right vertical axis is the cumulative percentage of the total valuation as shown by the cumulative valuation curve provided by the Pareto chart. The horizontal axis represents the spectrum size. For illustration purposes all bars are of an equal width meaning all users demand for an equal share of spectrum resources. Radio spectrum resources are of limited supply and the supply of the spectrum bandwidth is fixed at $\overline{S}$ (middle vertical line). If $\sum_{i=1}^{n} s_i \leq \overline{S}$, spectrum resources would not suffer from overuse, but if $\sum_{i=1}^{n} s_i > \overline{S}$, which is more likely, the spectrum would be overused if all users were allowed free access.

The social arbitrator in the proposed optimal model would allocate the spectrum usage to the first x users when the sum of the demand for the spectrum by these x users is equal to the fixed supply of the spectrum, i.e., $\sum_{i=1}^{x} s_i = \overline{S}$. Note the demand of user x can only be partially satisfied if the remaining spectrum size after serving the first $(x-1)$ users is less than s_x, which is the demand for the spectrum by user x. Such an allocation of spectrum resources is efficient as the social welfare of the spectrum usage can be maximized. Thus, the highest level of effectiveness the spectrum allocation can reach is $E_{optimum}$, representing the maximum percentage of social valuation that can be realized. If free markets could be well developed to coordinate the demand and supply of spectrum resources, market forces would drive the market price to the efficient level p^* such that only the first x users became the actual users of the spectrum, and the efficient usage of the spectrum would be realized. Therefore, the optimal regime balances all the three important but often conflicting objectives of spectrum management: full utilization, effective utilization and no interference, making the hybrid regime superior as it takes care of both spectrum utilization maximization and spectrum effectiveness maximization.

B. Case Study and Discussions

Although some access *cost* is required to differentiate users for the effective use of spectrum resources, the access cost has significant difference from existing auction-based dynamic spectrum market models that often focus on providing economic incentives for primary users to share their spectrum, and the auction outcomes resulting from profit maximization for the primary users are not necessarily maximizing social welfare. For example, a profit-driven primary user may not offer some spectrum for auction if doing so is not profitable due to transaction costs or competition concerns. In the proposed dynamic market component of the hybrid model, all unused or underused spectrum resources are supplied to the secondary market automatically, independent of primary users' decision-making[3]. The essential difference between these auction models is therefore the exclusion of the primary users from the dynamic reallocation of licensed spectrum. The maximum possible supply of licensed spectrum to the secondary market can be reached that helps reduce price and leads to more utilization. The cutoff price can be zero in the case of insufficient demand or sufficient supply of spectrum thus secondary users may access spectrum free of charge. On the other hand, if the spectrum access cost is fee-based, existing fee-based open access models require the fee be payable to primary users [17]. It has been suggested that license holders create "private commons" [18] allowing a form of unlicensed access which they charge for in some form. This is indeed an extreme case of dynamic spectrum markets where the license holder keeps an implicit contracts with any secondary user who agrees in advance to pay the license holder a fee for accessing the licensed band. Following previous analysis, economic incentives by primary users can deviate from social incentives, leading to unexploited opportunities when the equilibrium price is set above the zero-vacancy zero-interference optimal rate.

As a hypothetical case study to outline one possible scenario of the hybrid model, suppose a wireless carrier V wins Block B of X MHz FCC auction. User V is thus the primary user who has the prioritized right of accessing the block. As a precondition of the auction, the FCC rules are to allow dynamic access to the unused or underused block without V's permission. The FCC can use fine-designed standardized (time-region-block) spectrum packages to make an instantaneous market for trading residual Block B, analogous to real-time stock exchanges. The FCC supplies standardized packages (with the number of packages depending on the

[3]The supply of residual spectrum still depends on the primary user's actual use of the spectrum.

available residual spectrum) to the spot market, and secondary users compete for the packages in any time period. In this scenario, rather than acting as a broker, the government itself acts as the seller, enabling the maximum possible supply of spectrum bandwidth that leads to full utilization.

Specifically, considering the trading at time t, all secondary users know the trading price of a standard package (analogous to one share of stocks) and the total number of packages traded (equivalent to the supply of residual spectrum) in the previous time period $(t-1)$. Based on the information, secondary users send requests for the number of packages and a bid price of a standard package to the FCC. The FCC chooses the lowest bid price that would make the number of package demanded equal to the number supplied (or closest to but less than the number supplied) and sends the price back to secondary users. For example, if there are two packages available and the top two highest bids are p_i and p_j ($p_i < p_j$) sent by user i and j, then the price p_i would be the optimal cutoff price, paid by all secondary users with a bid no less than the price (i.e., user j who bid p_j only needs to pay p_i). In case that the total number of packages requested by secondary users is no higher than the number of packages available, the optimal price is set at zero. Note that unique in this model, rather than being self-interested, the government is a benevolent social optimizer seeking both full utilization and effective utilization of spectrum. Also note that the actual trade price is the cut-off price (i.e., the minimum price rather than the bid price) that not only can induce users to be honest in revealing their evaluation of spectrum but also reduces the set of information secondary users have to deal with.

Lastly, to avoid interference, the packages can have a term of termination: a secondary user's use of the spectrum can be automatically terminated if the primary owner requests access. Suppose at time $(t+1)$ the cognitive radio of user i senses signals sent by the primary user V, user i stops using the spectrum immediately and sends a new bandwidth request to the trade market which in turn assigns a different spectrum at a new price in real time. To improve *effectiveness*, it is the lower-bidding secondary users whose services are terminated first. For example, if user k bid less than user i, user k will be terminated and the bandwidth be reallocated to user i. The payment by secondary users for using the residual spectrum can be made at the end of each time period, based on the actual use of spectrum to accommodate the possibility of early termination.

IV. Conclusion

Wireless spectrum resources are limited, making how to efficiently utilize these limited resources an important topic. Cognitive radio technologies make an innovative step toward more utilization of spectrum. However, current models such as FCC auctions, cognitive radios, dynamic markets, or spectrum as commons alone cannot achieve an optimal solution. It is challenging to design a spectrum allocation regime that could satisfy all the three objectives, i.e., full utilization, effective utilization and zero interference at the same time. We argue the importance of social optimum from an economic perspective and make an initial attempt proving that a hybrid of dynamic spectrum market and dynamic spectrum access supported by cognitive radios can satisfy a good balance of the three objectives and can achieve the social optimum of wireless spectrum allocation. Dynamic sharing of spectrum is still in its infancy. Many complex issues in technical, economic, and regulatory aspects need to be addressed before its potential can be assessed and realized. It is our hope that our study can intrigue thinking and discussions in the design of optimal spectrum management regimes.

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