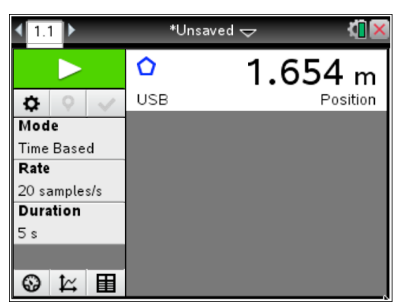
Lab 1: Zooming and Local Straightness

Laboratory Experience

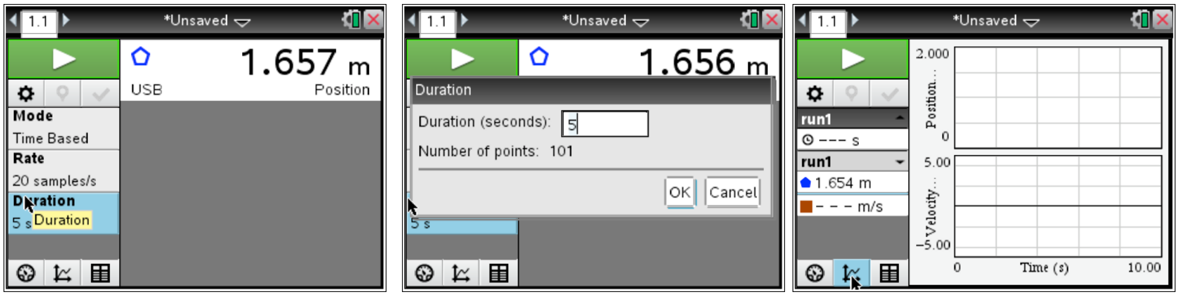
You have learned from previous classes that the slope of a non-vertical straight line can be obtained by looking at the ratio of the change in the *y*-coordinates to the change in the *x*-coordinates. In general we refer to this using the notation $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are any two points on the line. Up to now, most of the phenomena you have encountered regarding rates of change have probably been linear in nature. In Calculus we begin to study what it means for a function to have a rate of change when the function itself is not a straight line.

In this part of the lab you will need to use a Calculator-Based Ranger 2 (CBR2™) or other device that collects distance data in real-time. On the TI-Nspire CAS™ the data collection window (Vernier DataQuest application) opens when the CBR is connected to the calculator.

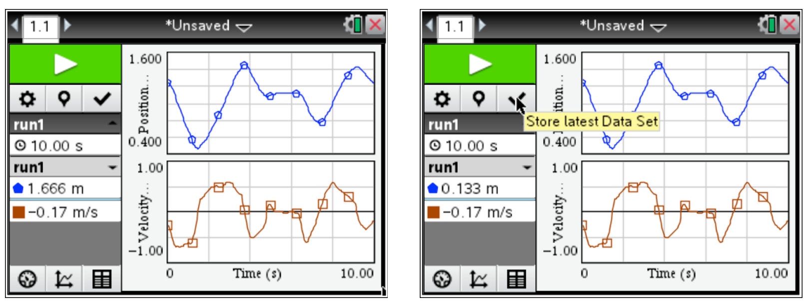
Up to now we have been looking at functions that can be expressed with algebraic symbols. However, not all functions in real life can be expressed so easily. In fact, more often we rely on data to help us describe some observable phenomenon. We can sometimes fit a nice curve to the data and then use the algebraic expression to tell us something about the function. However, this is not always possible. In this section of the lab we will use the Calculator-Based Ranger 2 (CBR2™) to collect position data as one of your group members walks in front of the device. As was demonstrated in class, we will open a data collection window on the TI-Nspire CAS™ using the Vernier DataQuest application which is fairly easy to use. You will collect data for a position-time graph in real time (10 seconds setting) as one of your group members walks in a straight line in front of the CBR2™ . To do this, open a new document on your handheld and then plug the CBR2 into the mini USB port. A data collection window will automatically open (see below).



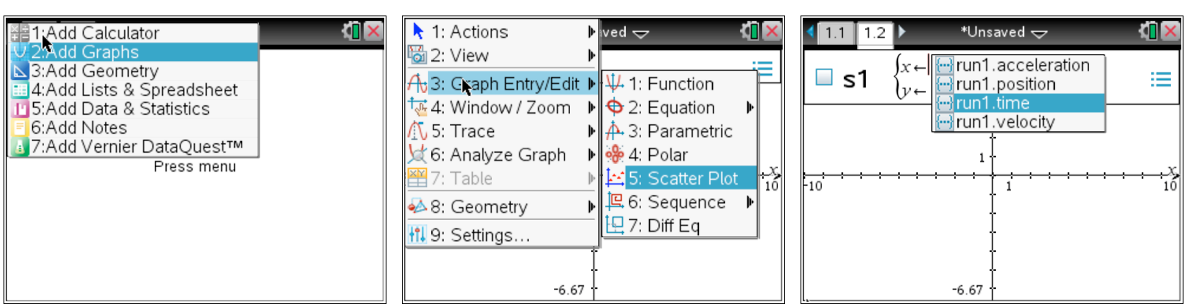
Now make sure the device is set to collect data for 10 seconds by clicking on the **Duration** menu and setting it to 10 seconds. Since we want to see the graph produced as your group member walks, click on the graph tab at the lower left corner of the window.



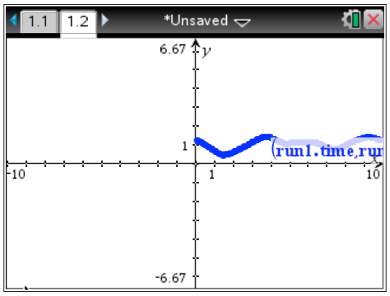
To begin data collection, click on the play button at the top left corner of the page and have one of your group members walk in a straight line back and forth in front of the CBR2™ . If you do not like the data from your experiment, simply repeat it until you have data you like. Once you have acceptable data, click the **Store Latest Data Set** icon (Check Mark) below the Play Button (see images below). This will store the data as “run1”.

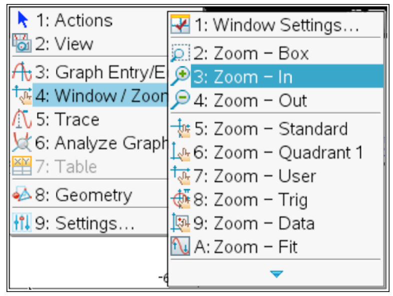


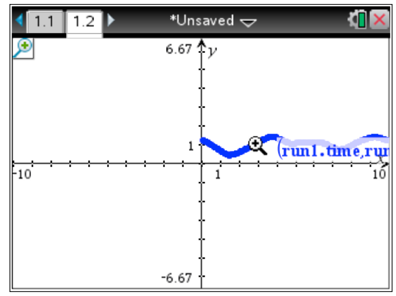
Since the zooming features within the **DataQuest** application are not as flexible, we will now graph your data in a regular Graphs page as a scatter plot. To do this, insert a new page by pressing ctrl +page and selecting **2:Graphs**. Now press the menu button and select **3:Graph Entry/Edit** followed by **5:Scatter Plot**. You will now see *x* and *y* entry lines where you can select the data you want plotted on the *x*- and *y*-axes. For this graph you want to have the time data on the *x*-axis and the position data on the *y*-axis. To easily access the names of the lists where these data are stored, just press the var button and all variables stored in the problem will appear in a list. Choose the appropriate data list names (in this case **run1.time** and **run1.position**) for the *x*- and *y*-axis respectively.



1. Zoom in on part of your graph. To do this, simply press menu and select **4:Window/Zoom** followed by **3:Zoom-In**. Now place the 🔍 icon over the portion of the graph you want to magnify and click.







a. What do you notice about the smoothness of the graph? What happens when you try to trace along the zoomed graph? How is this different from when you zoom and trace on a function graph on any basic graphing calculator?

b. Choose a point on the graph of the data where you would like to find the slope. Now choose a nearby point and calculate the slope through these two points. Show your calculation below. Thinking about the second point you chose for your calculation, explain why you chose it?

c. What does the slope you calculated represent physically? Think about the units it would have.

d. How could you get a more accurate approximation of the slope of your graph when performing the experiment?

Now that we have explored slope and its physical meaning through the use if discrete data, we can ask ourselves how this would play out with a continuous function. In this section of the lab we will use the same process of zooming to consider how we might define "slope" for a nonlinear function.

2. Graph the function $f(x) = x^4 - 5x^2 + 2x$ for $-3 \leq x \leq 3$.
   a. Zoom in on the point (2,0) until the graph looks like a straight line. Choose a point on the curve other than (2,0). Using the coordinates of the new point, calculate the slope of the straight line through these points. Show your calculation below. Give a rough sketch of your graph labeling the viewing window.

b. The number that you have calculated is an approximation of the actual slope of the curved graph at the point (2,0). Zoom out some and use this same method again to approximate the slope. Do you get the same answer as before? Again, sketch your graph and show your calculation below.

c. Now try zooming in several more times again and make the same slope calculation. Again, sketch your graph and show your calculation below. How does it compare to your previous calculations?

3. Use zooming to estimate the slope of the following functions at the given points. Give a rough sketch and viewing window for each. Explain why you think the estimate is accurate.

   a. $f(x) = x^3 - 2x + 5$ at $(-1,6)$

   b. $f(x) = (x-2)^{1/3}$ at $(3,1)$

   c. $f(x) = \sin(x)$ at $\left(\frac{\pi}{2},1\right)$

d. $f(x) = \sin(x)$ at (0,0)

Thus far in the lab you have used graphs of functions to estimate the *slope* of the function at a particular point. This slope is called *the derivative of the function* at the specified point. Although in these cases you have been able to estimate the slope, a function does not always have a slope at a specific point and thus has no derivative at the point.

4. Graph $f(x) = (x - 2)^{1/3}$ from part 3b again. Now zoom in on the point (2,0). Give a rough sketch of the graph and describe what you see as you zoom in closer and closer. Based on your graphical evidence, explain what you think the slope of f is at $x = 2$.

5. Graph $f(x) = \left| x^4 - 4x^2 \right|$ giving a rough sketch below. Based on your graphical evidence and zooming in on points chosen by your group, decide where the function has a derivative and where it does not. Support your conclusions with sketches and explanations.

6. When we consider the formal definition of the derivative of a function f at a point $(a, f(a))$ we get $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$. Explain in your own words/sketches/symbols how the process you used for approximating the slope of the graphs by repeated zooming throughout this lab is related to this formal definition of derivative. Be certain to explain the meaning of parts of the definition such as $a + h$ and $\lim_{h \to 0}$ as they are related to your experiences in the lab.