The Doomsday Rule

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The Doomsday rule is a method devised by John Conway for computing the day of the week of any given date. The method is based on first computing **Doomsday**, which is the day of the week of the last day of February.

First of all, we need a method of numbering the days of the week. We regard Sunday as Day 0, Monday as Day 1, Twosday as Day 2, etc.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
0	1	2	3	4	5	6

Note that the days of the week follow the rules of addition mod 7. For example, 4 days after Day 5 (Friday) is $4 + 5 \equiv 2 \bmod 7$, or Tuesday.

THE 20TH CENTURY

We will first consider dates in the 20th century; i.e., dates in the years 1900–1999. In 1900, the last day of February was a 3 (Wednesday). Since $365 \equiv 1 \bmod 7$, every ordinary year adds 1 day to Doomsday and every leap year adds 2 days. To get Doomsday for $1900 + Y$, we take

$$\text{Doomsday } 1900 + Y + [Y/4].$$

Here, $[Y/4]$ is the integer part of $Y/4$, or the number of leap years between 1900 and $1900 + Y$. For example, to get Doomsday 1929, we take

$$\text{Doomsday } 1900 + 29 + [29/4] = 3 + 29 + 7 = 39 \equiv 4 \bmod 7.$$

Thus February 28, 1929 was a Thursday.

There are a couple of tricks you can use if you want to do the calculations in your head. First, every dozen years advances Doomsday by one day since $12 + [12/4] = 12 + 3 = 15 \equiv 1 \bmod 7$. Thus if we want Doomsday 1969, we can observe that $69 = 5 \cdot 12 + 9$ and $[9/4] = 2$, so

$$\text{Doomsday } 1969 = 3 + 12 + 9 + 2 = 26 \equiv 5 \bmod 7 = \text{Friday.}$$

Secondly, 84 has 7 dozens, so Doomsday 1984 = Doomsday 1900 = 3. This makes the arithmetic simpler if you have a year between 1984 and 1999. For example, 1994 = 1984 + 10, and

$$\text{Doomsday } 1994 = \text{Doomsday } 1984 + 10 + 2 = 3 + 10 + 2 = 15 \equiv 1 \bmod 7 = \text{ Monday.}$$

Next, we need to know Doomsdays for every month. Here is one possible way of assigning the Doomsdays to the months.

Jan	31/32	*Jul*	11
Feb	28/29	*Aug*	8
Mar	7	*Sep*	5
Apr	4	*Oct*	10
May	9	*Nov*	7
Jun	6	*Dec*	12

For February, Doomsday is the 28th in an ordinary year and the 29th in a leap year. Similarly remember that for January, Doomsday is also the "last" day of the month– January 31 in an ordinary year and January 32 in a leap year. For the other months, Conway suggests using the following rules.

- For an even–numbered month such as April or June, Doomsday is the number of the month.

- For a long odd–numbered month, that is, an odd numbered month with 31 days, Doomsday is the number of the month plus 4. For example, May is the 5th month, May has 31 days, and Doomsday May$= 5 + 4 = 9$.

- For a short odd–numbered month, that is, an odd–numbered month with 30 days, Doomsday is the number of the month minus 4. For example, November is the 11th month, November has 30 days, and Doomsday November $= 11 - 4 = 7$.

Another way that Conway has suggested to help you remember the Doomsdays for odd numbered month is to remember the phrase "work from 9 to 5 at the 7-11." This should help to recall that the 9th is a Doomsday of the 5th month (May), and the 5th is a Doomsday for the 9th month (September). Similarly, the 7th is a Doomsday for the 11th month, and the 11th is the Doomsday for the 7th month. This takes care of all the odd-numbered months except March. For March, you can remember that March 0 is a Doomsday. (Most folks refer to March 0 as the last day of February.)

With this information, we can now compute the day of the week for any date between January 1, 1900 and December 31, 1999. The following examples illustrate.

May 24, 1968. May 9 is a Doomsday, so

$$\text{May } 24 = \text{Doomsday } + 15 = \text{Doomsday } + 1.$$

Now
$$\text{Doomsday } 1968 = 3 + 5 + 8 + 2 = 18 \equiv 4 \bmod 7,$$

so May 24, 1968 $= 4 + 1 = 5$, or Friday.

January 1, 1992. 1992 was a leap year, so January 32 = February 1 is a Doomsday. Now
$$\text{January } 1 = \text{Doomsday } - 31 = \text{Doomsday } + 4$$

since $-31 \equiv 4 \bmod 7$. Remembering that Doomsday 1984 = 3, we get

$$\text{Doomsday } 1992 = 3 + 8 + 2 = 13 \equiv 6 \bmod 7,$$

so January 1, 1992 $= 6 + 4 \equiv 3$, or Wednesday.

December 7, 1941. December 12 is a Doomsday, so

$$\text{December } 7 = \text{Doomsday } - 5 = \text{Doomsday } + 2.$$

Furthermore,
$$\text{Doomsday } 1941 = 3 + 3 + 5 + 1 \equiv 5 \bmod 7,$$

so December 7, 1941 $= 5 + 2 \equiv 0$, or Sunday.

OTHER A.D. DATES

Julius Caesar introduced the concept of the Leap Year; i.e., the idea of adding an extra day to February in every year divisible by 4. His calendar is called the *Julian Calendar*, and it was used throughout the Western world until 1582. In 1582, Pope Gregory XIII decreed a modification of the Julian Calendar. He declared that Century years (that is years divisible by 100) are leap years if and only if they are divisible by 400. Thus 1700, 1800, and 1900 are not leap years, but 2000 will be a leap year. The resulting calendar is called the *Gregorian Calendar*.

To apply the Doomsday rule for dates in other centuries, the only new piece of information you need is Doomsday for the Century year. The following table gives Doomsday for the Gregorian Calendar.

1500	1600	1700	1800
1900	2000	2100	2200
2300	2400	2500	2600
Wed	*Tue*	*Sun*	*Fri*

In practice, remember that 1900 was a Wednesday, and that each step backward to 1800, 1700, 1600 adds two days. Moreover, the Gregorian Calendar repeats itself every 400 years. Since March 19, 1994 was a Saturday, March 19, 1594 was also a Saturday, and March 19, 2394 will also be a Saturday.

February 22, 1809. February 28 is a Doomsday, so February 22 is Doomsday $-6 =$ Doomsday $+1$. Now

$$\text{Doomsday } 1809 = \text{Doomsday } 1800 + 9 + 2 = 5 + 9 + 2 \equiv 2 \bmod 7.$$

So February 22, 1809 $= 2 + 1 =$ Wednesday. (This is a famous birthday. Do you recognize it?)

January 1, 2000. January 32 is a Doomsday, because 2000 will be a leap year. Now Jan 1 $=$ Doomsday $- 31 =$ Doomsday $+ 4$. From the table above, Doomsday 2000 is Tuesday, so January 1, 2000 will be a Saturday.

In the Julian Calendar, Doomsday for the year $C \cdot 100$ is $6C$. For example,

$$\text{Doomsday } 1000 = 10 \cdot 6 = 60 \equiv 4 \bmod 7.$$

Therefore Doomsday 1066 $= 4 + 5 + 6 + 1 \equiv 2 \bmod 7$. The Battle of Hastings was fought on October 14, 1066 $=$ Doomsday $+ 4 = 2 + 4 = 6$–a Saturday.

The Gregorian reform was adopted by omitting the dates October 5–14, 1582. Thus you should use the Gregorian Calendar for dates on or after October 15, 1582, and you should use the Julian Calendar for dates on or before October 4, 1582. Catholic countries such as Italy, France, and Spain adopted the Gregorian Calendar immediately, but Britain and the American colonies did not switch until 1752. In making the switch, they omitted the days September 3–13, 1752. Sweden made the switch by omitting leap days in the years 1700–1740.

To calculate a B.C. date, add a big enough multiple of 28 or 700 years to turn it into an A.D. date. Remember that there was no year 0, so you will have to add 1 to compensate. For example, the day of Creation, according to Archbishop Ussher, was October 23, 4004 B.C. To get Doomsday 4004 B.C., we add $700 \cdot 6 + 1 = 4201$, giving

$$\text{Doomsday 4004 B.C.} = \text{Doomsday 197} = 6 + 13 + 3 \equiv 1 \bmod 7.$$

Since October 23 = Doomsday $+ 13 = 1 + 13 \equiv 0 \bmod 7$, October 23, 4004 B.C. was a Sunday.

The B.C.–A.D. system for numbering years was created by a sixth-century Roman monk named Dionysius Exiguus. The notion of zero was not introduced into Europe until the 13th century; consequently, Dionysius did not allow for a year 0. Furthermore, Dionysius erred in his reckoning of Christ's birth; subesquent historical scholarship has shown that Christ was born somewhere between 5 and 3 B.C.

COMPUTING EASTER

Easter is the first Sunday strictly later than the Paschal full moon, which is an arithmetical approximation to the astronomical one. The Paschal full moon is given by the formula

$$(\text{April 19} = \text{March 50}) - (11G + C) \bmod 30,$$

except that when the formula gives

- April 19 you should take April 18,

and when the formula gives

- April 18 and $G \geq 12$, you should take April 17.

In the formula, G is the Golden number. For the year Y, the Golden number is

$$G = Y \bmod 19 + 1.$$

The term C is the Century term. For all Julian years, $C = 3$. In the Gregorian calendar,

$$C = \begin{cases} -4 & \text{for } 15xx, 16xx \\ -5 & \text{for } 17xx, 18xx, . \\ -6 & \text{for } 19xx, 20xx. \end{cases}$$

The general formula for C in the Gregorian year Hxx is

$$-H + [H/4] + [8(H + 11)/25].$$

You can use the Doomsday rule to get the day of the week of the Paschal full moon, then you can easily get Easter by finding the next Sunday.

Let's work out Easter 1994 in detail. First of all,

$$G = 1994 \bmod 19 + 1 = 18 + 1 = 19.$$

The century term is -6 for all years of the form $19xx$, so the Paschal full moon is on March

$$50 - (11 \cdot 19 - 6) \bmod 30 = 50 - 23 = 27.$$

Now March 27 is Doomsday $+ 20 =$ Doomsday $- 1$, and Doomsday 1994 is Monday. Therefore the Paschal Full Moon is on Sunday, and Easter is the following Sunday, April 3.

Note that in the formula for the Paschal Full Moon, the term $(11G + C) \bmod 30$ will always be at least 0. If this term is 0, then the Paschal Full Moon will be April 18. If this date should happen to fall on a Sunday, then Easter will be on April 25, which is the latest that Easter can happen in a given year.

Computing Rosh Hashana

Rosh Hashana is the Jewish New Year's Day. In a Gregorian year Y A.D. (i.e., the first day of the Jewish year $Y + 3761$), it happens on September N, where

$$\{[Y/100] - [Y/400] - 2\} + \frac{765433}{492480}(12G)_{\bmod 19} + \frac{1}{4}(Y)_{\bmod 4} - \frac{313Y + 89091}{98486} = N + \text{ fraction}$$

and G is the Golden number, except that it must be postponed by one or two days in the following circumstances.

- If September N is a Sunday, Wednesday, or Friday, then it must be postponed to the following day.

- If September N is a Monday,

$$\text{fraction} \geq \frac{23269}{25920} \text{ and } (12G)_{\bmod 19} > 11,$$

  then it must be postponed to the following day.

- If September N is a Tuesday,

$$\text{fraction} \geq \frac{1367}{2160} \text{ and } (12G)_{\bmod 19} > 6,$$

  then it must be postponed to the following Thursday (*not* the following Wednesday.)

For Julian years, the computation is the same except that the term $\{[Y/100]-[Y/400]-2\}$ is omitted.

Reference

This material is adapted from *Winning Ways for Your Mathematical Plays* by E.R. Berlekamp, J. H. Conway, and R.K. Guy. This two–volume work, published by Academic Press, covers a number of topics in recreational mathematics. The Doomsday rule is discussed on pages 795–797 of Volume 2.

Doomsday Homework

1. Find the day of the of week of your birthdate.

2. Find the day of the of week of the birthdate of some member of your family other than yourself.

3. The space shuttle Challenger exploded on January 28, 1986. What day of the week was it?

4. There was exactly one Saturday between November 10, 1966 and November 16, 1966. Which day was it?

5. What day of the month will Thanksgiving be on this year? (**Hint:** In the United States, Thanksgiving occurs on the fourth Thursday of November.)

6. The "Star Trek: Deep Space Nine" episode entitled "Past Tense" is set in the 21st century. In one scene, a calendar shows the date as Friday, August 30, 2030. Is the day of week correct?

7. In the movie "Demolition Man", the date August 3, 2032, is identified as a Monday. Is this correct?

8. The movie "The Gunslinger" has been featured on "Mystery Science Theater 3000." In the movie, the words "Friday, May 21, 1878" are imposed on the opening scene. Did the scriptwriter get the day of the week correct?

9. In the book *Fried Green Tomatoes at the Whistle Stop Cafe*, an elderly woman named Virginia Threadgoode recalls events from her distant past. On page 12, she states, "Some people thought it started the day she met Ruth, but I think it started that Sunday dinner, April the first, 1919, the same year Leona married John Justice." Explain why this recollection is in error.

10. Let us assume that Virgina Threadgoode was correct about the day of the week (Sunday) and the day of the year (April 1) in the passage in the previous problem. If so, she incorrectly remembered the year. Find a year between 1915 and 1921 when April 1 occurred on a Sunday.

11. When I was preparing this handout, I went through some old files on recreational mathematics. In one of them, I found a poster announcing a talk that I gave to the Math Club on the Doomsday Rule. The poster gives the date of the talk as Tuesday, January 13. It does not give the year. My memory isn't good enough to recall the year I gave this talk; however, I am certain that it happened between 1985 and 1990. Help me figure out what year it was.

12. In the book *October 1964*, David Halberstam chronicles the 1964 major league baseball

season. The book is very well written, but there are some minor errors in the accounts of the last week of the pennant races. On page 299, he writes, "On Sunday, September 29, Tony Kubek had a frustrating day at the plate ... ." On page 307, he writes, "On Monday, September 28, the Phillies went into St. Louis for a three-game series." At least one of these dates is clearly wrong. Which one?

13. Martin Luther King delivered his famous "I have a dream" speech on August 28, 1963. What day of the week was it?

14. The Declaration of Independence was signed on July 4, 1776. What day of the week was it?

15. Find the day of the of week of the bicentennial of the Declaration of Independence.

16. Columbus "discovered" America on October 12, 1492. What day of the week was it?

17. Shakespeare and Cervantes both died on April 23, 1616. Yet Shakespeare died on Tuesday and Cervantes died on Saturday. Explain.

18. The first expedition to circumnavigate the Earth was headed by Magellan. The expedition started with 265 sailors and five ships; it finished with 18 sailors and one ship. Pigafetta, one of the surviving sailors, kept a careful diary throughout the voyage. The first date was marked "September 20, 1519." What day of the week was it?The return to Spain was marked "September 6, 1522." What day of the week was it?

19. When Pigafetta returned, the Spainards on shore told him he had lost track of one day–that it was September 7, not September 6. Pigafetta insisted that he kept careful records, and, furthermore, one other member of the crew had a log that agreed with Pigafetta. Is there an alternate explanation for the discrepancy?

20. A man was nearly 48 years old on celebrating his first birthday. Where, when, and what day of the week was it?

21. According to legend, the ancient Greek philosopher Thales predicted that there would be a total eclipse of the Sun on May 28, 585 B.C. What day of the week was that?

22. In the early morning hours of June 17, 1972, four men were arrested while breaking into Democratic Party headquarters in the Watergate complex in Washington, D.C. What day of the week was it?

23. On the night of August 8, 1974, Richard M. Nixon announced that he would resign the Presidency. The resignation became official on the next day. What day of the week was the latter?

24. Here are some famous Civil War dates. Compute the day of the week of each one.

   (a) Confederates fire on Fort Sumter: April 20, 1861.

(b) The first day of the Battle of Gettysburg: July 1, 1863

(c) Lee surrenders at Appomatox: April 10, 1865.

25. Each of the following days is the date of the assasination of some historical figure. Find the day of the week and identify the person who was assasinated.

(a) April 14, 1865.

(b) November 22, 1963.

(c) April 4, 1968.

(d) June 5, 1968.

(e) March 15, 44 B.C.

26. During a Monday night football game in early December, 1980, Howard Cosell announced to a nationwide audience that John Lennon had just been assasinated. What day was it?

27. Sarah Bernhardt appeared at the Calumet Theater on May 30, 1911. What day of the week was it?

28. Compute the date of Easter for next year.

29. Explain why Easter never occurs earlier than March 21.

30. (Hard.) Find a year when Easter occurs on April 25.

31. World War I started on August 4, 1914, and ended on November 11, 1918. Find the day of the week of these dates.

32. On April 8, 1974, Hank Aaron hit the 715th home run of career, and thereby broke a major league record that had been held by Babe Ruth. What day of the week was it?

33. On May 26, 1959, Harvey Haddix, a pitcher for the Pittsburgh Pirates, pitched twelve perfect innings against the Milwaukee Braves. Despite this amazing performance, he lost the game in the 13th inning. What day of the week was it?

34. World War II was started when Germany invaded Poland on September 1, 1939. What day of the week was it?

35. Fermat announced his "little theorem" in a letter dated October 18, 1640. What day of the week was it?

36. Andrew Wiles first announced that he had proved Fermat's Last Theorem on June

23, 1994. On December 4, 1993, he posted an e-mail message stating that a gap had been found in his proof. On October 7, 1994, he claimed that he had fixed the gap in the proof and released two new manuscripts, one of which was co-authored with Richard Taylor. Find the day of the week of each of these dates.

37. The last day of the Julian Calendar was October 4, 1582. What day of the week was it?The first day of the Gregorian Calendar was October 15, 1582. What day of the week was it?

38. According to legend, Cleopatra committed suicide on August 30, 30 B.C. What day of the week was it?