Math 132 - Quiz 4

Each of the following can be answered with a calculator method accompanied by a good guess. However, you must show the details of a solution which uses *our calculus method.*

1. Evaluate $\lim_{x\to 0} \frac{1-e^{3x}+3x}{x^2}$

   Use L'Hospital's Rule as long as it applies.

   $$\lim_{x\to 0} \frac{1-e^{3x}+3x}{x^2} = \lim_{x\to 0} \frac{-3e^{3x}+3}{2x} = \lim_{x\to 0} \frac{-9e^{3x}}{2} = \frac{-9\lim_{x\to 0} e^{3x}}{\lim_{x\to 0} 2} = \frac{-9\cdot 1}{2} = -\frac{9}{2}$$

2. Given $f(x) = 3x^4 + 8x^3$,

   (a) The interval(s) on which f is increasing and the interval(s) on which f is decreasing.

   (b) The x-coordinate(s) of any local extreme points (indicating as max or min).

   $f'(x) = 12x^3 + 24x^2 = 12x^2(x+2)$ so 0 and -2 are critical numbers.

	$12x^2$	$x+2$	$f'(x)$	f
$x < -2$	+	−	−	dec.
$-2 < x < 0$	+	+	+	inc.
$0 < x$	+	+	+	inc.

   So f is decreasing on $(-\infty, 2)$ and increasing on $(-2, 0)$ and $(0.\infty)$.

   f has a local minimum at -2.

   (c) The interval(s) on which f is concave up and the interval(s) on which f is concave down.

   (d) The x-coordinate(s) of any inflection points.

   $f''(x) = 36x^2 + 48x = 12x(3x+4)$ so 0 and $-4/3$ are boundaries of interest.

	$12x$	$3x+4$	$f''(x)$	f
$x < -4/3$	−	−	+	concave up
$-4/3 < x < 0$	−	+	−	concave down
$0 < x$	+	+	+	concave up

   So f is concave up on $(-\infty, -\frac{4}{3})$ and $(0, \infty)$ and concave down on $(-\frac{4}{3}, 0)$

   Inflection points are at $x = -\frac{4}{3}$ and $x = 0$.

3. Use Newton's Method to find the *negative solution* of $(x-2)e^x = -1$, showing:

   (a) The initial approximation that leads to your solution

   $x_0 = -1$ (Others possible.)

   (b) Your next four approximations (labeled) following the one in (a)

   $x_1 = -1.140859086$, $x_2 = -1.146185641$, $x_3 = -1.146193221$,$x_4 = -1.146193221$

   (c) Your solution to 6 decimal places.

   -1.146193