Math 132 - Quiz 3

1. Suppose that 200 individuals of the species, "Rodents of Unusual Size", are planted on a large, uninhabited island. Also suppose that, 4 months later, the population count is 220 individuals. Assuming that the population can be modeled with an exponential growth model:

a Find a formula for the population count at any time t (in months).

Use $P = Ce^{kt}$ where t is time (in months) and P is the number of rodents. Then $P(0) = 200$ and hence $200 = Ce^{k\cdot 0} = C$ so we have $P = 200e^{kt}$.

Then $220 = 200e^{k\cdot 4}$ which implies $\ln 1.1 = 4k$ or $k = .25 \ln 1.1$. The function becomes $P = 200e^{(.25\ln 1.1)t}$

b Using your result of part (a), find the number of months required for the population to double the original count.

We are to find t so that $P(t) = 400$. Solve $400 = 200e^{(.25\ln 1.1)t}$ giving $\ln 2 = .25\ln 1.1)t$ and $t = \frac{4\ln 2}{\ln 1.1} \approx 29.09$ months.

2. Find the derivative of the function, $f(x) = \ln|2 - x - 5x^2|$

$$f'(x) = \frac{1}{2 - x - 5x^2}\frac{d}{dx}(2 - x - 5x^2) = \frac{-1 - 10x}{2 - x - 5x^2}$$

3. Find the derivative of the function, $f(x) = \cos^{-1}(3 - x^2)$

$$f'(x) = -\frac{1}{\sqrt{1 - (3 - x^2)^2}}\frac{d}{dx}(3 - x^2) = -\frac{1}{\sqrt{1 - (3 - x^2)^2}}(-2x) = \frac{2x}{\sqrt{1 - (3 - x^2)^2}}$$

4. Find the derivative of the function, $f(x) = \cosh(\ln x)$. Give the answer in a completely simplified form (i.e. no logarithms, exponentials, trig functions, etc.).

Method 1:

$$\cosh x = \frac{e^x + e^{-x}}{2} \text{ so } \cosh(\ln x) = \frac{e^{\ln x} + e^{-\ln x}}{2} = \frac{e^{\ln x} + \frac{1}{e^{\ln x}}}{2} = \frac{x + \frac{1}{x}}{2} = \frac{1}{2}(x + x^{-1})$$

Then $f'(x) = \frac{1}{2}(1 - x^{-2})$

Method 2

$$\cosh x = \frac{e^x + e^{-x}}{2} \text{ so } \frac{d}{dx}\cosh x = \frac{e^x - e^{-x}}{2}$$

$$\text{Hence, } f'(x) = \frac{e^{\ln x} - e^{-\ln x}}{2}\frac{d}{dx}\ln x = \frac{e^{\ln x} - \frac{1}{e^{\ln x}}}{2}\frac{1}{x} = \frac{x - \frac{1}{x}}{2}\frac{1}{x} = \frac{1}{2}(1 - x^{-2})$$